S. Supplementary data

This supplement contains additional figures and tabular data from the numerical experiments.

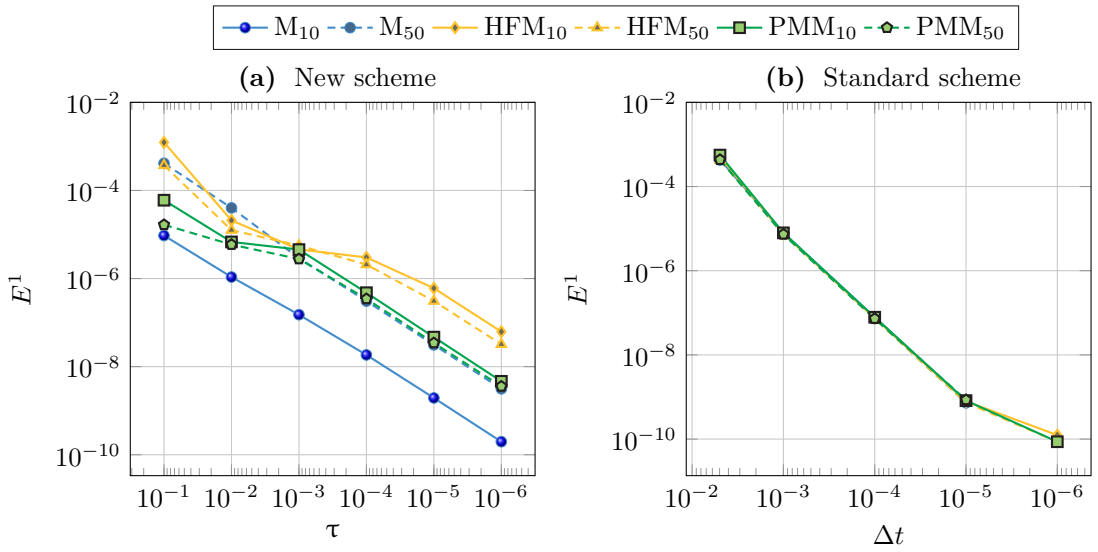

FIGURE S1. L^1-error against reference solution (new scheme with $\tau = 10^{-9}$) in the source-beam test ($n_x = 600, t_{\text{end}} = 2.5$). (a) New scheme for decreasing tolerance parameter τ. (b) Standard scheme for decreasing time step Δt.

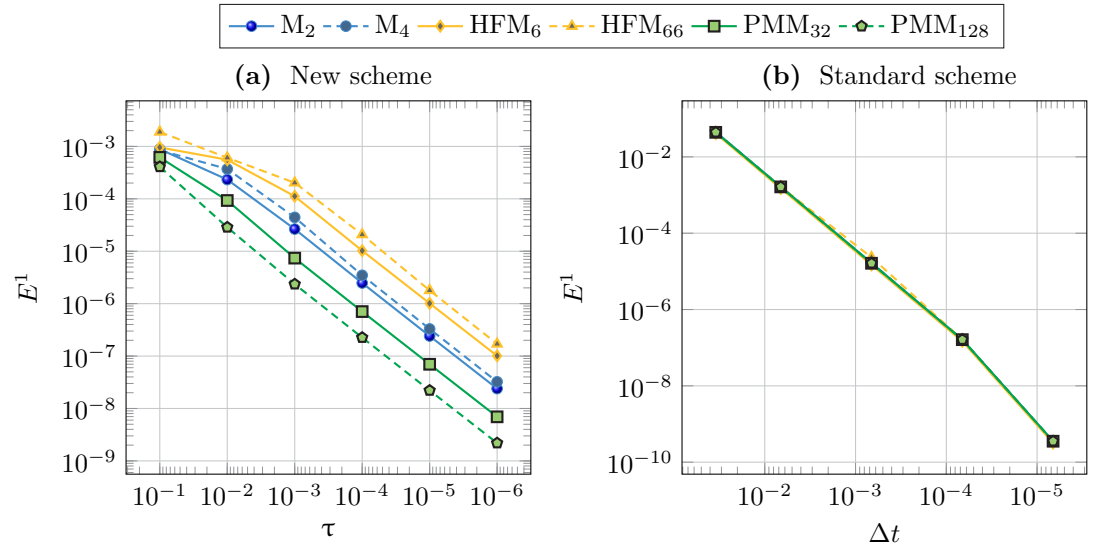

FIGURE S2. L^1-error against reference solution (new scheme with $\tau = 10^{-9}$) in the point-source test ($n_{\mathbf{x}} = 30^3, t_{\text{end}} = 0.25$). (a) New scheme for decreasing tolerance parameter τ. (b) Standard scheme for decreasing time step Δt.

Test case	n_x	t_{end}	Model	Scheme	Δt	τ	E^1	E^∞
Plane-source	1200	1	HFM_2	new	—	1e-03	1.47e-04	1.63e-04
Plane-source	1200	1	HFM_2	standard	0.001800	—	4.24e-03	9.33e-03
Plane-source	1200	1	HFM_{10}	new	—	1e-03	2.89e-04	1.97e-04
Plane-source	1200	1	HFM_{10}	standard	0.001800	—	4.27e-03	5.69e-03
Plane-source	1200	1	HFM_{50}	new	—	1e-03	2.40e-04	1.07e-04
Plane-source	1200	1	HFM_{50}	standard	0.001800	—	2.79e-04	9.40e-04
Plane-source	1200	1	HFM_{100}	new	—	1e-03	2.43e-04	1.07e-04
Plane-source	1200	1	HFM_{100}	standard	0.001800	—	2.61e-04	9.42e-04
Plane-source	1200	1	PMM_2	new	—	1e-03	8.80e-05	9.68e-05
Plane-source	1200	1	PMM_2	standard	0.001800	—	4.24e-03	9.33e-03
Plane-source	1200	1	PMM_{10}	new	—	1e-03	2.83e-05	5.52e-05
Plane-source	1200	1	PMM_{10}	standard	0.001800	—	3.94e-03	5.53e-03
Plane-source	1200	1	PMM_{50}	new	—	1e-03	1.66e-05	1.38e-05
Plane-source	1200	1	PMM_{50}	standard	0.001800	—	4.42e-04	8.76e-04
Plane-source	1200	1	PMM_{100}	new	—	1e-03	1.55e-05	1.23e-05
Plane-source	1200	1	PMM_{100}	standard	0.001800	—	2.65e-04	9.40e-04
Plane-source	1200	1	M_1	new	—	1e-03	8.79e-05	9.67e-05
Plane-source	1200	1	M_1	standard	0.001800	—	4.24e-03	9.33e-03
Plane-source	1200	1	M_{10}	new	—	1e-03	3.00e-05	3.77e-05
Plane-source	1200	1	M_{10}	standard	0.001800	—	4.61e-03	4.58e-03
Plane-source	1200	1	M_{50}	new	—	1e-03	4.61e-05	3.63e-05
Plane-source	1200	1	M_{50}	standard	0.001800	—	3.22e-04	9.43e-04
Plane-source	1200	1	M_{100}	new	—	1e-03	5.13e-05	3.13e-05
Plane-source	1200	1	M_{100}	standard	0.001800	—	2.66e-04	9.43e-04
Source-beam	1200	2.5	HFM_2	new	—	1e-03	1.83e-04	5.52e-04
Source-beam	1200	2.5	HFM_2	standard	0.002250	—	2.19e-05	8.27e-04
Source-beam	1200	2.5	HFM_{10}	new	—	1e-03	2.57e-05	8.08e-05
Source-beam	1200	2.5	HFM_{10}	standard	0.002250	—	2.49e-05	8.31e-04
Source-beam	1200	2.5	HFM_{50}	new	—	1e-03	7.60e-07	6.42e-07
Source-beam	1200	2.5	HFM_{50}	standard	0.002250	—	2.23e-05	8.35e-04
Source-beam	1200	2.5	HFM_{100}	new	—	1e-03	1.14e-06	1.01e-06
Source-beam	1200	2.5	HFM_{100}	standard	0.002250	—	2.23e-05	8.35e-04
Source-beam	1200	2.5	PMM_2	new	—	1e-03	2.43e-04	2.24e-03
Source-beam	1200	2.5	PMM_2	standard	0.002250	—	2.21e-05	8.27e-04
Source-beam	1200	2.5	PMM_{10}	new	—	1e-03	5.60e-05	3.17e-04
Source-beam	1200	2.5	PMM_{10}	standard	0.002250	—	2.72e-05	8.26e-04
Source-beam	1200	2.5	PMM_{50}	new	—	1e-03	1.59e-07	1.26e-07
Source-beam	1200	2.5	PMM_{50}	standard	0.002250	—	2.25e-05	8.26e-04
Source-beam	1200	2.5	PMM_{100}	new	—	1e-03	1.74e-07	1.51e-07
Source-beam	1200	2.5	PMM_{100}	standard	0.002250	—	2.24e-05	8.26e-04
Source-beam	1200	2.5	M_1	new	—	1e-03	3.29e-04	6.00e-03
Source-beam	1200	2.5	M_1	standard	0.002250	—	2.21e-05	8.27e-04
Source-beam	1200	2.5	M_{10}	new	—	1e-03	2.21e-07	1.54e-06
Source-beam	1200	2.5	M_{10}	standard	0.002250	—	2.95e-05	8.20e-04
Source-beam	1200	2.5	M_{50}	new	—	1e-03	5.35e-07	4.62e-07
Source-beam	1200	2.5	M_{50}	standard	0.002250	—	2.28e-05	8.26e-04
Source-beam	1200	2.5	M_{100}	new	—	1e-03	1.60e-06	1.55e-06
Source-beam	1200	2.5	M_{100}	standard	0.002250	—	2.24e-05	8.26e-04

TABLE S1. L^1/L^∞ errors compared to reference solution (new scheme with $\tau = 10^{-6}$) for the one-dimensional test cases.

Test case	$n_{\mathbf{x}}$	t_{end}	Model	Scheme	Δt	τ	E^1	E^∞
Point-source	50^3	0.75	M_1	new	—	1e-02	1.35e-04	6.56e-05
Point-source	50^3	0.75	M_1	standard	0.020785	—	1.62e-02	1.24e-02
Point-source	50^3	0.75	M_2	new	—	1e-02	1.36e-04	1.13e-04
Point-source	50^3	0.75	M_2	standard	0.020785	—	1.56e-02	1.45e-02
Point-source	50^3	0.75	M_3	new	—	1e-02	1.68e-04	8.16e-05
Point-source	50^3	0.75	M_3	standard	0.020785	—	1.40e-02	9.21e-03
Point-source	50^3	0.75	M_4	new	—	1e-02	1.74e-04	7.35e-05
Point-source	50^3	0.75	M_4	standard	0.020785	—	1.30e-02	7.88e-03
Point-source	50^3	0.75	HFM_6	new	—	1e-02	2.97e-04	5.92e-04
Point-source	50^3	0.75	HFM_6	standard	0.020785	—	1.27e-02	2.43e-02
Point-source	50^3	0.75	HFM_{18}	new	—	1e-02	2.72e-04	2.34e-04
Point-source	50^3	0.75	HFM_{18}	standard	0.020785	—	1.34e-02	1.48e-02
Point-source	50^3	0.75	HFM_{66}	new	—	1e-02	2.86e-04	1.51e-04
Point-source	50^3	0.75	HFM_{66}	standard	0.020785	—	1.30e-02	8.56e-03
Point-source	50^3	0.75	PMM_{32}	new	—	1e-02	4.37e-05	2.84e-05
Point-source	50^3	0.75	PMM_{32}	standard	0.020785	—	1.31e-02	8.02e-03
Point-source	50^3	0.75	PMM_{128}	new	—	1e-02	1.24e-05	1.14e-05
Point-source	50^3	0.75	PMM_{128}	standard	0.020785	—	1.30e-02	7.66e-03
Checkerboard	70^3	3.2	M_1	new	—	1e-02	4.50e-05	3.67e-06
Checkerboard	70^3	3.2	M_1	standard	0.051962	—	2.10e-02	2.35e-02
Checkerboard	70^3	3.2	M_2	new	—	1e-02	5.10e-05	7.04e-06
Checkerboard	70^3	3.2	M_2	standard	0.051962	—	2.36e-02	3.27e-02
Checkerboard	70^3	3.2	M_3	new	—	1e-02	4.54e-05	6.06e-06
Checkerboard	70^3	3.2	M_3	standard	0.051962	—	2.43e-02	2.85e-02
Checkerboard	70^3	3.2	PMM_{32}	new	—	1e-02	4.29e-05	5.52e-06
Checkerboard	70^3	3.2	PMM_{32}	standard	0.051962	—	2.41e-02	2.93e-02
Checkerboard	70^3	3.2	HFM_6	new	—	1e-02	1.14e-04	1.54e-05
Checkerboard	70^3	3.2	HFM_6	standard	0.051962	—	2.25e-02	2.75e-02
Checkerboard	70^3	3.2	HFM_{18}	new	—	1e-02	1.19e-04	1.48e-05
Checkerboard	70^3	3.2	HFM_{18}	standard	0.051962	—	2.39e-02	2.95e-02
Shadow	18000	20	M_1	new	—	1e-02	1.34e-08	3.84e-09
Shadow	18000	20	M_1	standard	0.103923	—	4.32e-02	9.14e-03
Shadow	18000	20	M_1	standard	0.040000	—	1.12e-02	5.28e-03
Shadow	18000	20	M_2	new	—	1e-02	5.98e-10	1.25e-10
Shadow	18000	20	M_2	standard	0.103923	—	3.40e-02	1.06e-02
Shadow	18000	20	M_2	standard	0.040000	—	8.94e-03	4.42e-03
Shadow	18000	20	PMM_{32}	new	—	1e-02	2.07e-10	7.18e-11
Shadow	18000	20	PMM_{32}	standard	0.103923	—	3.30e-02	9.87e-03
Shadow	18000	20	PMM_{32}	standard	0.040000	—	8.77e-03	4.39e-03
Shadow	18000	20	PMM_{128}	new	—	1e-02	1.29e-10	4.14e-11
Shadow	18000	20	PMM_{128}	standard	0.103923	—	3.27e-02	9.77e-03
Shadow	18000	20	PMM_{128}	standard	0.040000	—	8.72e-03	4.39e-03
Shadow	18000	20	HFM_6	new	—	1e-02	3.42e-08	4.82e-09
Shadow	18000	20	HFM_6	standard	0.103923	—	2.75e-02	7.22e-03
Shadow	18000	20	HFM_6	standard	0.040000	—	7.61e-03	3.55e-03
Shadow	18000	20	HFM_{18}	new	—	1e-02	1.05e-09	2.73e-10
Shadow	18000	20	HFM_{18}	standard	0.103923	—	3.04e-02	9.69e-03
Shadow	18000	20	HFM_{18}	standard	0.040000	—	8.30e-03	4.31e-03
Shadow	18000	20	HFM_{66}	new	—	1e-02	2.61e-10	9.21e-11
Shadow	18000	20	HFM_{66}	standard	0.103923	—	3.29e-02	9.69e-03

TABLE S2. L^1/L^∞ errors compared to reference solution (new scheme with $\tau = 10^{-6}$) for the three-dimensional test cases.

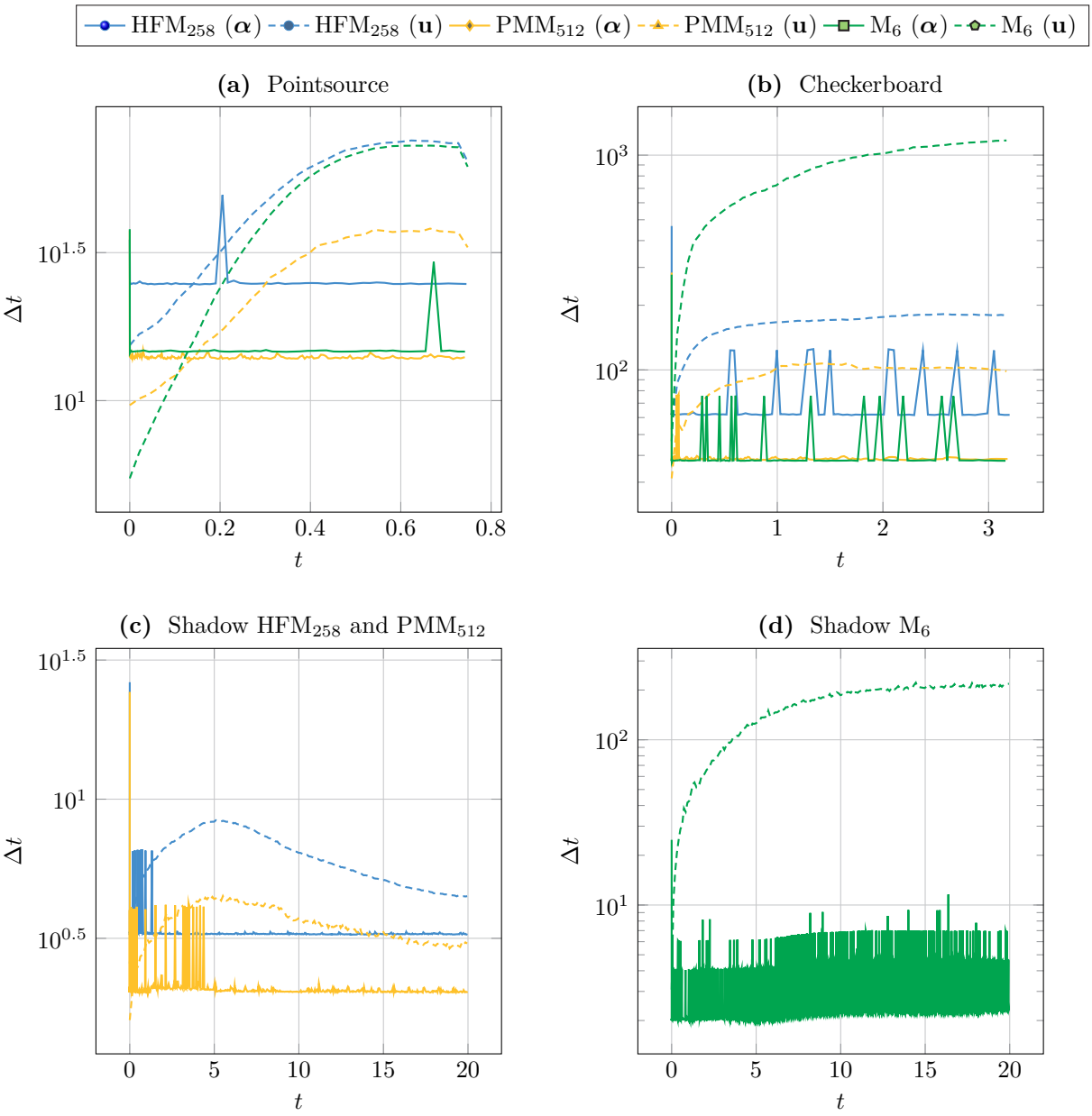

FIGURE S3. Wall times for computing a single time step. (a) Point-source problem, $n_{\mathbf{x}} = 50^3$, $t_{\text{end}} = 0.75$. (b) Checkerboard problem, $n_{\mathbf{x}} = 70^3$, $t_{\text{end}} = 3.2$. (c) Shadow problem, $n_{\mathbf{x}} = 60 \times 20 \times 15$, $t_{\text{end}} = 20$, HFM_{258} and PMM_{512}. (d) Shadow problem, M_6.

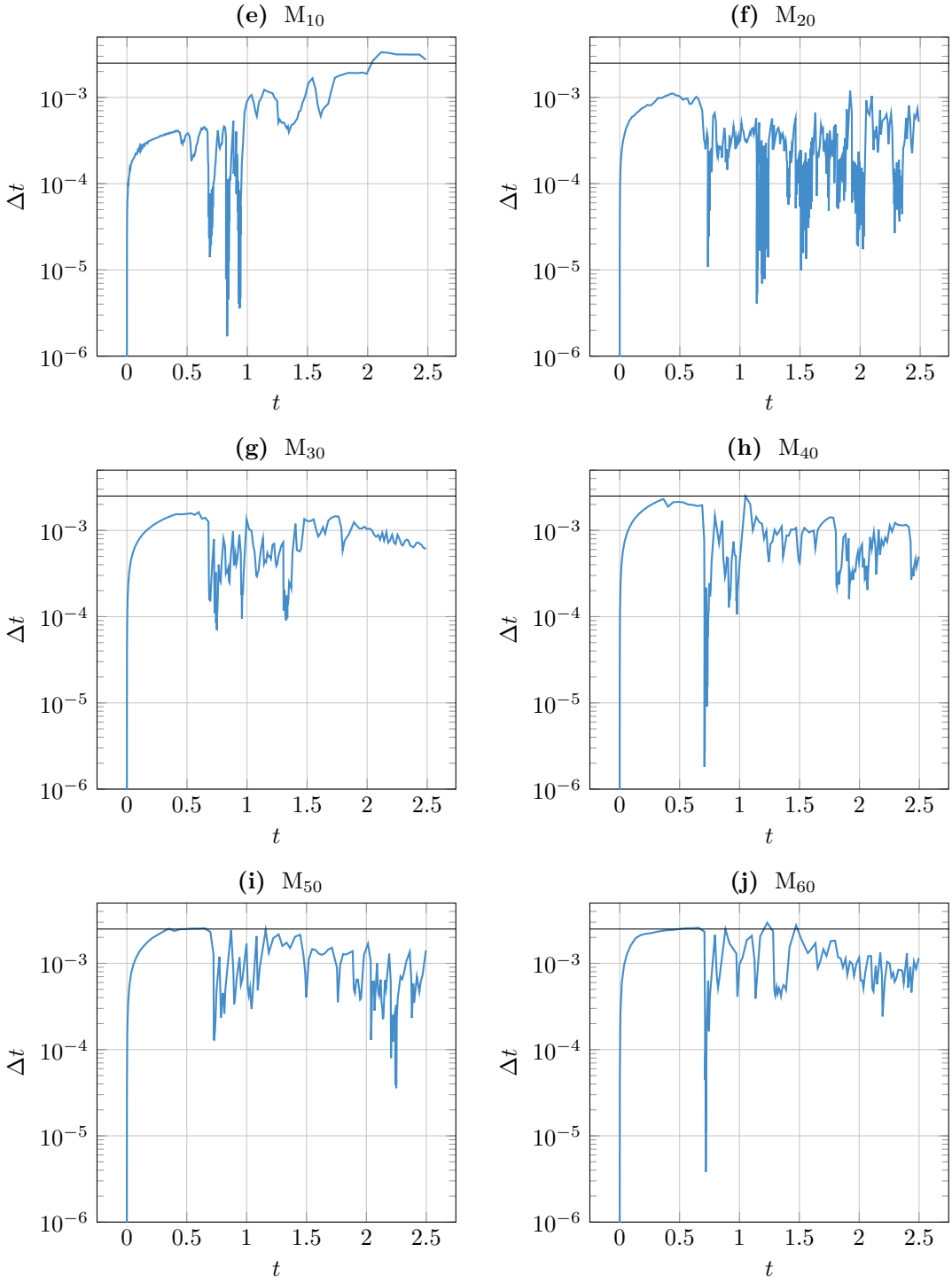

FIGURE S4. Time steps taken in the source-beam test case ($n_x = 1200$, $t_{\text{end}} = 2.5$, $\tau = 10^{-3}$) for different models. The solid line represents the maximum realizability preserving time step $\Delta t_{\max}$ for the standard splitting scheme.

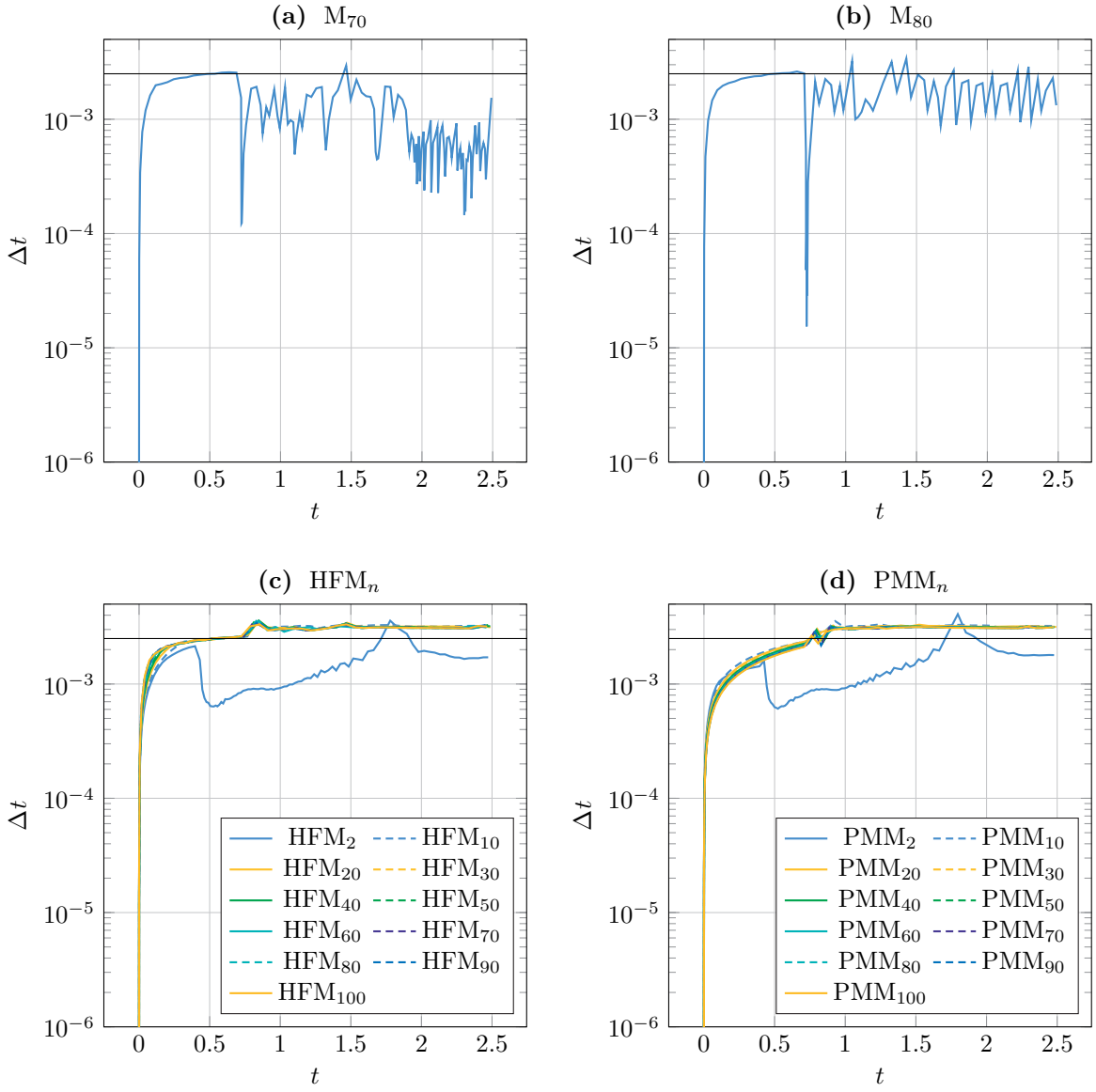

FIGURE S5. Time steps taken in the source-beam test case ($n_x = 1200$, $t_{\text{end}} = 2.5$, $\tau = 10^{-3}$) for different models (continued). The solid line represents the maximum realizability preserving time step $\Delta t_{\max}$ for the standard splitting scheme.

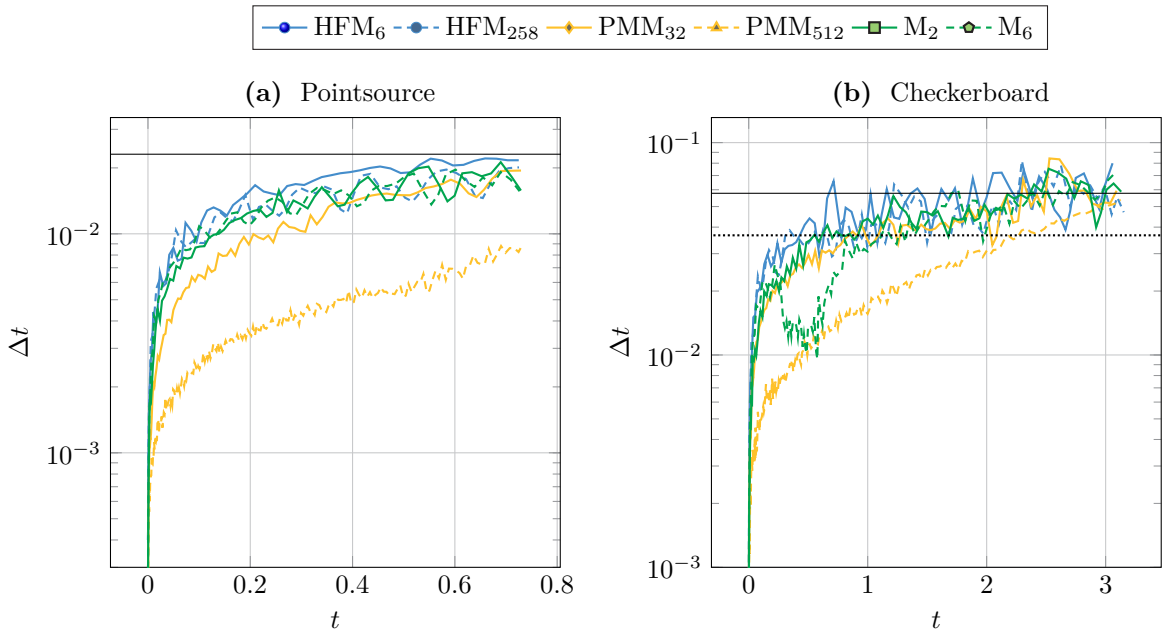

FIGURE S6. Time steps taken in the point-source and checkerboard tests ($\tau = 0.01$). The solid and dotted horizontal line represent the time step restrictions (2.37) and (2.34), respectively (which almost agree for the point-source test). (a) Point-source problem with 50^3 grid cells, $t_{\text{end}} = 0.75$. (b) Checkerboard problem with 70^3 grid cells, $t_{\text{end}} = 3.2$.

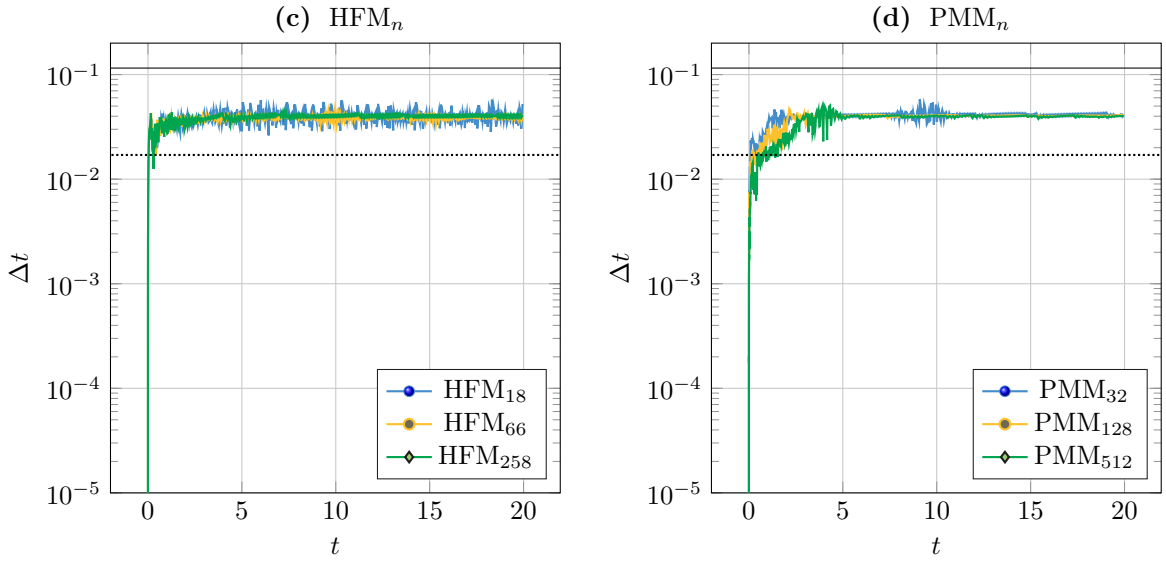

FIGURE S7. Time steps taken in the shadow test case ($n_{\mathbf{x}} = 60 \times 20 \times 15$, $t_{\text{end}} = 20$, $\tau = 10^{-2}$) for the HFM$_n$ and PMM$_n$ models. The solid and dotted horizontal line represent the time step restrictions (2.37) and (2.34), respectively.

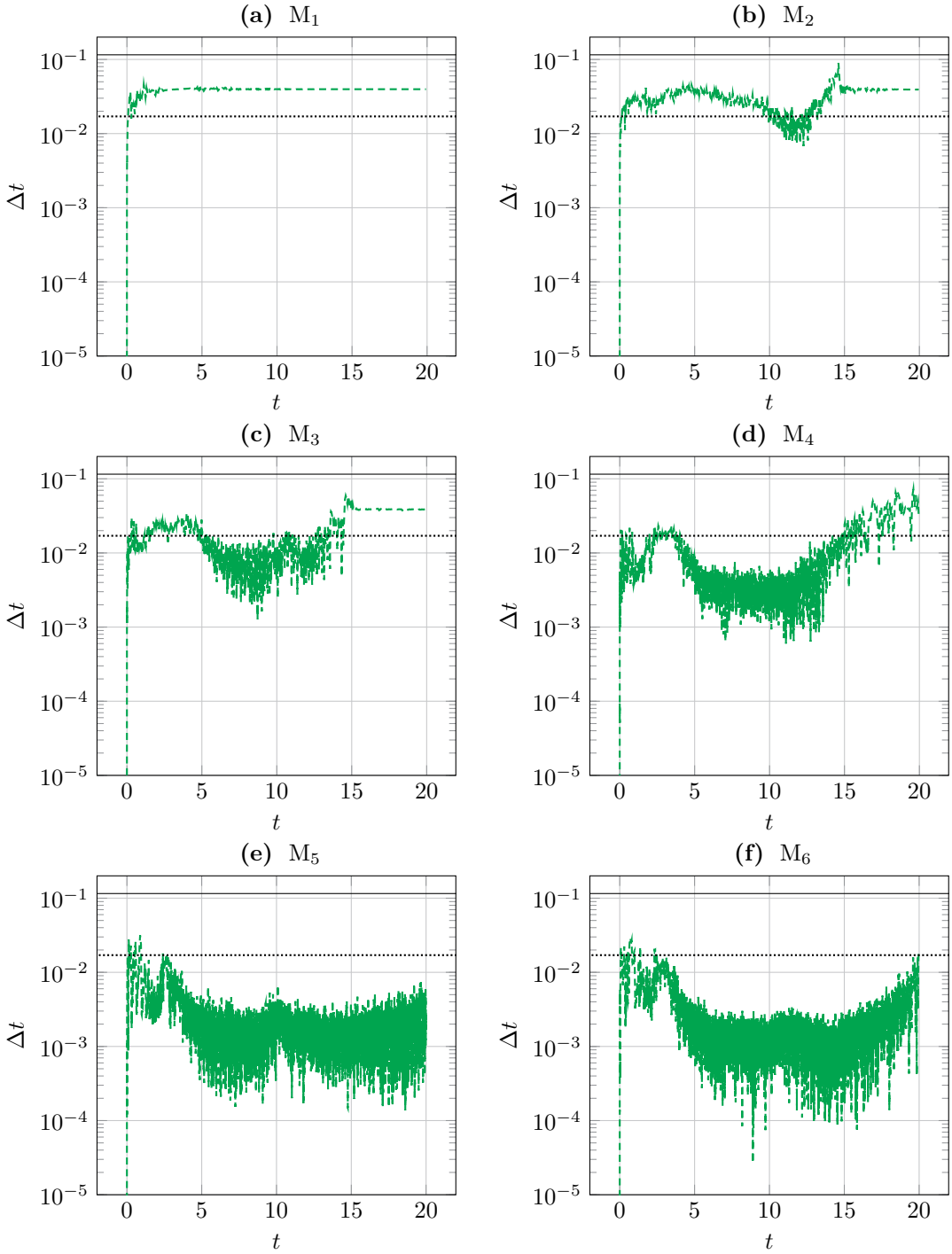

FIGURE S8. Time steps taken in the shadow test case ($n_{\mathbf{x}} = 60 \times 20 \times 15$, $t_{\text{end}} = 20$, $\tau = 10^{-2}$) for the M_N models. The solid and dotted horizontal line represent the time step restrictions (2.37) and (2.34), respectively.

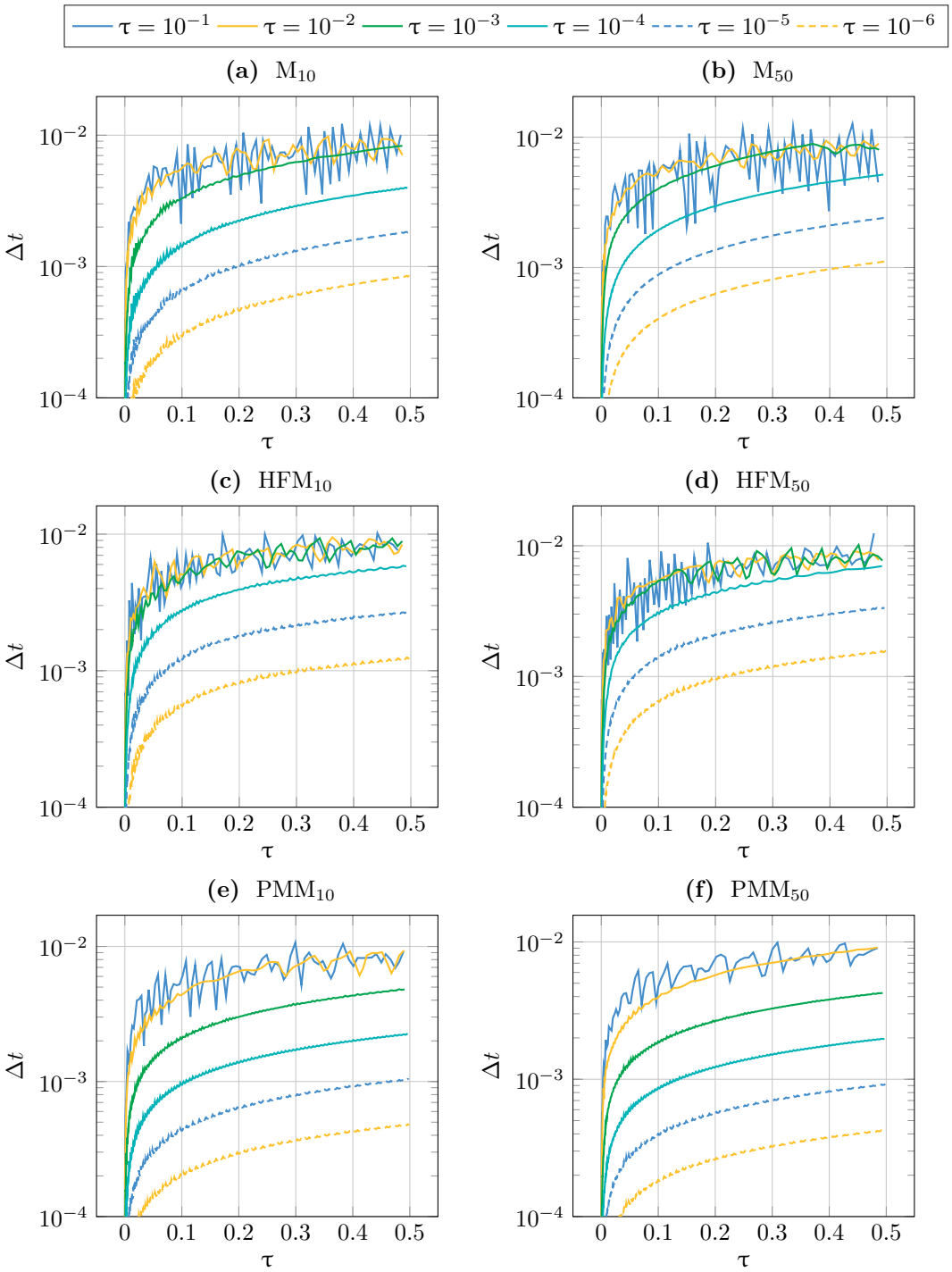

FIGURE S9. Time steps taken in the plane-source test case ($n_x = 240$, $t_{\text{end}} = 0.5$) for different tolerance parameters τ. The last step has been omitted for all models as it was chosen to reach t_{end} exactly and thus may be artificially small.

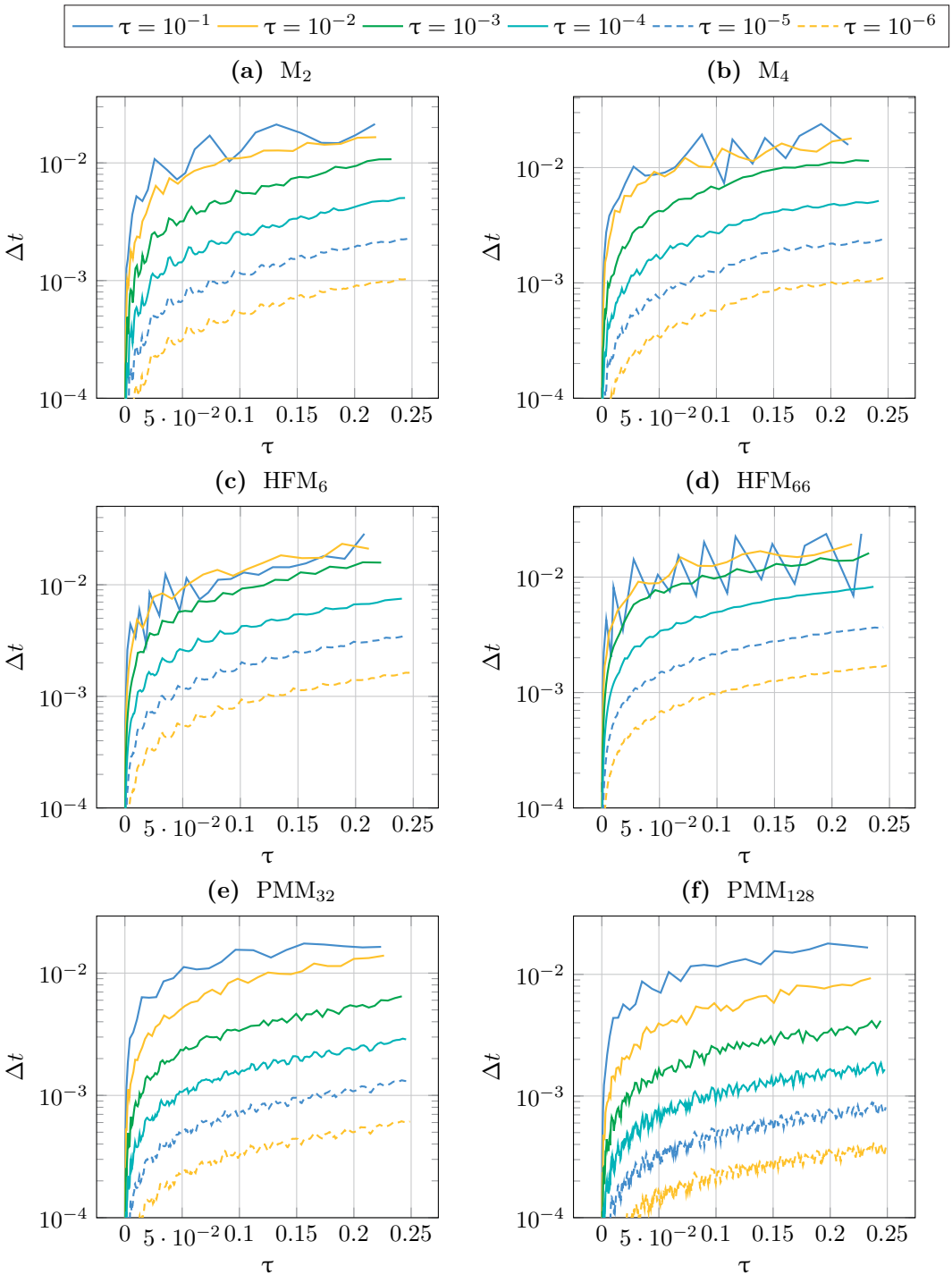

FIGURE S10. Time steps taken in the point-source test case ($n_{\mathbf{x}} = 30^3$, $t_{\text{end}} = 0.25$) for different tolerance parameters τ. The last step has been omitted for all models as it was chosen to reach t_{end} exactly and thus may be artificially small.