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DIFFUSION APPROXIMATION OF NONLINEAR ELECTRON PHONON COLLISION MECHANISMS (*)

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Abstract. — *We study the behaviour of the solutions of the Boltzmann equation of solid state physics when the mean free path tends to zero. The nonlinear Boltzmann operator considered in this paper models collisions between electrons and phonons with constant energy. Taking into account degeneracy effects we show that distribution functions relax to local equilibrium functions* similar to *Fermi Dirac distributions depending on energy dependent chemical potentials, which are periodic with period equal to the phonon energy. In the fluid limit these chemical potentials solve an energy dependent drift diffusion equation.*

Résumé. — *On étudie le comportement asymptotique des solutions de l'équation de Boltzmann des semiconducteurs quand le libre parcours moyen tend vers zéro. L'opérateur quadratique de Boltzmann considéré modélise les collisions entre électrons et phonons d'énergie constante. En prenant en compte les effets de dégénérescence, nous montrons que les fonctions de distributions tendent vers des fonctions correspondant à des équilibres locaux. Ces fonctions sont semblables à des distributions de Fermi-Dirac avec des potentiels chimiques qui sont des fonctions périodiques de l'énergie. Dans la limite fluide, ces potentiels chimiques résolvent une équation de dérive-diffusion où l'énergie intervient comme une variable indépendante.*

1. INTRODUCTION

In semiconductor kinetic theory the distribution of electrons $f = f(t, x, v)$ is assumed to solve the following Boltzmann equation

$$\frac{\partial}{\partial t} f + v \cdot \nabla_x f - \frac{q}{m_\star} E \cdot \nabla_v f = Q(f), t > 0, x \in IR^3, v \in IR^3 . \qquad (1.1)$$

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The variables t, x, v represent the time, the position and the velocity of particles respectively. The constants q and $m_\star$ are the charge and the effective mass of electrons. The electric field $E = E(t, x)$ depends only on time and positions. For the sake of simplicity it will be assumed to be known for the following. Also we have assumed that the parabolic band approximation holds. We refer to [MRS, P1] for the physical background and details concerning equation (1.1). The by now classical scaling of the equation (1.1) leads to

$$\alpha^2 \frac{\partial}{\partial t} f^\alpha + \alpha(v \, . \, \nabla_x f^\alpha - E \, . \, \nabla_v f^\alpha) = Q(f^\alpha) ,$$

$$t > 0 , \quad x \in IR^3 , \quad v \in IR^3 . \tag{1.2}$$

The integral operator Q is quadratic and acts only on the variable v. It reads

$$Q(f) = \int_{R^3} [f' a^{-|v|^2} (1 - f) - f a^{-|v'|^2} (1 - f')] \, \sigma(v, v') \, dv' \tag{1.3}$$

where $a > 1$ is given.

Here $f'$and f stand for $f(v')$, $f(v)$. The kernel σ is a positive symmetric measure.

The fluid limit of (1.2) is carried out by taking the limit of f^α when $\alpha \to 0$. When σ is smooth and positive, then the kernel of Q is only spanned by Fermi-Dirac distributions

$$F(\mu, |v|^2) = \frac{1}{1 + a^{|v|^2 - \mu}} , \quad \mu \in (-\infty, \infty] \tag{1.4}$$

and has nice comptactness properties. Using this fact it is shown in [GP] (see also [PS]) that the distribution function f^α tends to a Fermi-Dirac distribution $F(\mu(t, x), v)$. The chemical potential $\mu(t, x)$ solves a non-linear drift diffusion equation. But the situation is different for singular cross sections σ. In this paper we consider a cross section which models collisions between electrons and phonons with a constant energy which is equal to 1 after the performed scaling. We have

$$\sigma(v, v') = G(v, v') \, [a^{|v|^2} \delta(|v|^2 - |v'|^2 + 1) + a^{|v'|^2} \delta(|v|^2 - |v'|^2 - 1)] , \tag{1.5}$$

where G is a smooth symmetric positive function.

In this case Majorana [Maj1] and [Maj2] proved that the kernel of Q consists of $F(\mu(|v|^2), |v|^2)$ where μ is any 1-periodic function. We shall see that the behaviour of the fluid limit in this case is completely different from the *regular* case.

This paper follows the analysis of [MS] where the linear case is studied. There weak compactness is enough to pass to the limit $\alpha \to 0$ and a convergence proof is given in [MS]. The lack of compactness of Q when σ is given by (1.5) does not allow to use the technics of [GP] in the nonlinear case. Therefore we restrict ourselves to the formal derivation of the fluid limit.

The paper is organized as follows. The next Section is devoted to the basic properties of the collision operators. In Section 3, the fluid approximation is derived. Finally, in Section 4, we study perturbations of (1.2) by operators of the form $\varepsilon^n Q_1$ where the intersection of the nullspaces of Q and Q_1 consists only of Fermi-Dirac distributions. For $n = 1$ we recover the usual drift diffusion model and for $n = 2$ a relaxation term appears as a source term in the energy dependent drift-diffusion equation.

2. BASIC PROPERTIES OF THE COLLISION OPERATOR

Physically relevant distribution functions satisfy $0 \leqslant f \leqslant 1$. Therefore we introduce the space

$$V = \{f \in C^0(IR^3) \,;\, 0 \leqslant f \leqslant 1\} \tag{2.1}$$

and define

$$N(Q) = \{f \in V \,;\, Q(f) = 0\}$$

$$P_1 = \{p \in C^0(IR^+ \,;\, IR) \,;\, p \text{ is } 1-\text{periodic}\}\,. \tag{2.2}$$

We have

PROPOSITION 2.1 : *[Maj2] Let Q be the operator defined by (1.3), (1.5) then*
1. (invariants) for any $p \in P_1$ and any $f \in V$

$$\int_{IR^3} Q(f)\,(v)\, p(|v|^2)\, dv = 0 \tag{2.3}$$

2. (entropy estimate) let χ be an increasing function. Then for any $f \in V$ and $h = f a^{|v|^2} / (1 - f)$

$$-\int Q(f)\,\chi(h)\, dv = \frac{1}{2}\int (h - h')\,[\chi(h) - \chi(h')] \times$$

$$\times (1 - f)(1 - f')\, a^{-|v|^2 - |v'|^2}\, \sigma\, dv\, dv' \geqslant 0 \tag{2.4a}$$

3. (local equilibrium) the inequality (2.4a) becomes an equality iff $f \in N(Q)$ *and*

$$N(Q) = \{F(\mu(|v|^2), |v|^2) ; \mu \in P_1\} \tag{2.4b}$$

where F is defined by (1.4).

In Section 4 we need the following generalization.

PROPOSITION 2.2 : *Let Q be given by (1.3) and let* σ *be any positive symmetric measure. Then (2.4a) holds and we have an equality iff* $f \in N(Q)$.

Proof : (2.4a) is just a computation using the symmetry of σ. Now, if this inequality is an equality it means that $(h - h')(1 - f)(1 - f') = 0$ on the support of σ. Therefore

$$Q(f)(v) = \int (h - h')(1 - f)(1 - f') a^{-|v|^2 - |v'|^2} \sigma \, dv' = 0 \quad \bullet$$

Let Q_1 be the collision operator with kernel

$$\sigma_1(v, v') = G_1(v, v') [a^{|v|^2} \delta(|v|^2 - |v'|^2 + r) + a^{|v'|^2} \delta(|v|^2 - |v'|^2 - r)]$$

for same $r > 0$ and $G_1 > 0$ symmetric in v, v'. Then $N(Q + Q_1) = N(Q) \cap N(Q_1)$.

Obviously $N(Q) \cap N(Q_1) = \{F(\mu, |v|^2) ; \mu \in (-\infty, \infty]\} =: N_0$ iff r is irrational. This corresponds to a certain instability of the nullspaces of operators defined by (1.3), (1.5).

In Section 3 we also need properties of the linearization of Q around $f_0 \in N(Q)$. Denoting by L_{f_0} this linearization, we have

$$L_{f_0}(g)(v) = \int (s_{f_0}(v', v) g' - s_{f_0}(v, v') g) \, dv',$$

$$s_{f_0}(v', v) := \sigma(v, v') [a^{-|v|^2}(1 - f_0) - f_0 a^{-|v'|^2}] . \tag{2.5}$$

We want to write L_{f_0} in a symmetric form.

First we use that if $f_0(v) = F(\mu_0(|v|^2), v)$, $\mu_0 \in P_1$ we have $a^{-|v|^2}(1 - f_0) = a^{-\mu_0(|v|^2)} f_0$. Then

$$s_{f_0}(v', v) = \sigma(v, v') f_0 [a^{-\mu_0(|v|^2)} + a^{-|v'|^2}] .$$

But on the support of σ we have $|v'|^2 = |v|^2 \pm 1$. Therefore $\mu_0(|v|^2) = \mu_0(|v'|^2)$ and we obtain

$$s_{f_0}(v', v) = \sigma(v, v')\, a^{-\mu_0(|v|^2)} f_0[1 + a^{\mu_0(|v'|^2) - |v'|^2}]$$

$$= \sigma(v, v')\, a^{-\mu_0(|v|^2)} f_0\left[1 + \frac{f_0'}{1 - f_0'}\right]$$

$$= \sigma(v, v')\, a^{-\mu_0(|v|^2)} f_0 f_0' \frac{1}{f_0'(1 - f_0')} .$$

Setting $\sigma_{f_0}(v, v') = \sigma(v, v')\, a^{-\mu_0(|v|^2)} f_0(v) f_0(v')$ we see that σ_{f_0} is symmetric and we can rewrite (2.5)

$$L_{f_0}(g) = \int \sigma_{f_0}\left(\frac{g'}{f_0'(1 - f_0')} - \frac{g}{f_0(1 - f_0)}\right) dv . \tag{2.6}$$

PROPOSITION 2.3 : *The nullspace of L_{f_0} is*

$$N_{f_0} = \{q(|v|^2) f_0(1 - f_0) ; q \in P_1\} . \tag{2.7}$$

For any function $p \in P_1$ and g in $C^0(IR)^3$ we have

$$L_{f_0}(pg) = pL_{f_0}(g) \tag{2.8}$$

$$\int L_{f_0}(g)\, p\, dv = 0 . \tag{2.9}$$

Proof: Multiplying (2.6) by $q = g/f_0(1 - f_0)$ and integrating we obtain $q(v) = q(v')$ on the support of σ. It follows that q depends only on $|v|^2$ and is 1-periodic w.r.t. this variable. To obtain (2.8) we have just to remark that $p(|v'|^2) = p(|v|^2)$ on the support of σ. Then (2.9) is a consequence of (2.8) and of the symmetry of the expression (2.6). •

In Section 3 we need to solve equations of the type $L_{f_0}(g) = h$. For that we perform explicit computations. We set

$$v = \sqrt{c}\Omega , \quad |\Omega| = 1 , \quad c = |v|^2 .$$

Then we have the following

LEMMA 2.1 : *If the function G is of the form*

$$G(v, v') = \overline{G}(c, c', \Omega . \Omega') > 0 \tag{2.10}$$

then

$$\int \Omega' G(v, v')\, d\Omega' = H(c, c')\, \Omega$$

$$\int G(v, v')\, d\Omega' = I(c, c')$$

and

$$|H(c, c')| < I(c, c') .$$

Proof: Without loss of generality we can assume that $\Omega = (0, 0, 1)$. The result is immediate using spherical coordinates. •

In order to avoid technical difficulties, from now on we assume that there exist positive constants λ, $I_\star$, $I^\star$ such that

$$|\tilde{H}(c)| < \lambda \tilde{I}(c)\,, \quad \lambda < 1 \tag{2.11}$$

$$0 < I_\star \leqslant \tilde{I}(c) \leqslant I^\star < \infty \tag{2.12}$$

where $\tilde{H}(c) = H(c, c+1)$ $\tilde{I}(c) = I(c, c+1)$. Then we have

PROPOSITION 2.4 : *For* $\mu_0 \in P_1$ *let* $f_0(c) = F(\mu_0(c), c)$. *Then there is a function* $g(\mu_0, c)$ *such that*

$$- L_{f_0}(g(\mu_0(|v|^2), |v|^2)) = v f_0(|v|^2)\,(1 - f_0(|v|^2)) . \tag{2.13a}$$

Moreover

$$g(\mu_0(c), c) \geqslant 0\,, \quad \int_0^\infty a^c\, g(\mu_0(c), c)\, dc < \infty . \tag{2.13b}$$

Proof: In the following we use the convention that a function taken at $c < 0$ vanishes. We set $h(c) = g(\mu_0(c), c)/f_0(1 - f_0)$. From (1.5), (2.6) and Lemma 2.1 we obtain

$$- L_{f_0}(g(\mu_0(|v|^2), |v|^2)) =$$

$$= (\alpha(c)\, h(c) - \beta(c)\, h(c-1) - \gamma(c)\, h(c+1))\, v \tag{2.14}$$

with

$$\alpha(c) = \frac{a^{c-\mu_0}}{2} f_0(c) \, [f_0(c+1) \, \tilde{I}(c) + a^{-1} f_0(c-1) \, \tilde{I}(c-1)]$$

$$\beta(c) = \frac{a^{c-\mu_0}}{2} f_0(c) \, a^{-1} f_0(c-1) \, \tilde{H}(c-1)$$

$$\gamma(c) = \frac{a^{c-\mu_0}}{2} f_0(c) f_0(c+1) \, \tilde{H}(c) \, .$$

Then (2.13) reduces to that three stage recursion

$$\begin{aligned} &[f_0(c+1) \, \tilde{I}(c) + a^{-1} f_0(c-1) \, \tilde{I}(c-1)] \, h(c) \\ &- a^{-1} f_0(c-1) \, \tilde{H}(c-1) \, h(c-1) - f_0(c+1) \, \tilde{H}(c) \, h(c+1) \\ &= 2 \, a^{\mu_0 - c}(1 - f_0(c)) = 2 f_0(c) \, . \end{aligned} \tag{2.15}$$

This recursion is of the type studied in [MS]. The technics used there lead to (2.13). •

3. FLUID APPROXIMATION

We begin to derive the conservation laws corresponding to (1.2) where the collision operator is defined by (1.3) and (1.5). The energy dependent concentrations and fluxes are defined according to

$$n^\alpha(t, x, c) = \int f^\alpha(t, x, v) \, d\Omega \, \frac{\sqrt{c}}{2} \tag{3.1}$$

$$j^\alpha(t, x, c) = \frac{1}{\alpha} \int f^\alpha(t, x, v) \, \Omega \, d\Omega \, \frac{c\sqrt{c}}{2} \, . \tag{3.2}$$

Then the usual concentrations and fluxes are computed by :

$$N^\alpha(t, x) = \int_0^\infty n^\alpha(t, x, c) \, dc \, , \quad J^\alpha(t, x) = \int_0^\infty j^\alpha(t, x, c) \, dc \, . \tag{3.3}$$

Let $p \in P_1$. We multiply (1.2) by $p(|v|^2)$ and integrate with respect to v. Using (2.3) we obtain

$$\partial_t \int_0^\infty n^\alpha \, p \, dc + \operatorname{div} \int_0^\infty j^\alpha \, p \, dc - \frac{1}{\alpha} E \cdot \int_0^\infty \nabla_v f^\alpha \, p \, dv = 0 \, . \tag{3.4}$$

Integrating by parts gives

$$\frac{1}{\alpha}\int \nabla_v f^\alpha \, p \, dv = -\frac{1}{\alpha}\int 2\, v f^\alpha \, p'(v^2)\, dv$$

$$= -2\int_0^\infty j^\alpha \, p'(c)\, dc = 2\int_0^\infty \frac{\partial}{\partial c} j^\alpha \, p \, dc$$

because $j^\alpha(c=0) = j^\alpha(c=\infty) = 0$. In this way we obtain

$$\int_0^\infty \left[\partial_t n^\alpha + \mathrm{div}_x j^\alpha - 2\, E \cdot \frac{\partial}{\partial c} j^\alpha\right] p \, dc = 0$$

$$\text{for any } p \in P_1 . \tag{3.5}$$

This conservation law has now to be completed by an equation of state connecting n^α and j^α. We formally expand f^α in power of α

$$f^\alpha = f_0 + \alpha f_1 + o(\alpha) . \tag{3.6}$$

Inserting (3.6) in (1.2) gives

$$Q(f_0) = 0 \tag{3.7}$$

$$-L_{f_0}(f_1) = -v \,.\, \nabla_x f_0 + E \,.\, \nabla_v f_0 . \tag{3.8}$$

The operator L_{f_0} is the linearization of Q around f_0 given by (2.5). We obtain from (2.4b)

$$f_0 = f_0(t, x, c) = F(\mu_0(t, x, c), c) , \quad \mu_0(t, x, .) \in P_1 . \tag{3.9}$$

It follows from (3.1), (3.6) and (3.9) that

$$n^\alpha = n^0(\mu_0(t, x, c), c) + o(1) \tag{3.10}$$

$$n^0(\mu_0, c) = 2\,\pi \sqrt{c}\, F(\mu_0, c) . \tag{3.11}$$

In order to compute the flux, we first remark that

$$\int \Omega f_0 \, d\Omega = f_0 \int \Omega \, d\Omega = 0 .$$

Therefore we obtain

$$j^{\alpha} = j_1 + o(1), \quad j_1 = \frac{c\sqrt{c}}{2}\int \Omega f_1 \, d\Omega . \tag{3.12}$$

Our goal is to compute j_1 in terms of μ_0. Therefore we have to solve (3.8). But if f_0 is given by (3.9), (3.8) can be rewritten :

$$-L_{f_0}(f_1) = v f_0(1-f_0)\left(-\nabla_x \mu_0 - E + E \cdot \frac{\partial \mu_0}{\partial c}\right). \tag{3.13}$$

Note that $\left(-\nabla \mu_0 - E + E \cdot \frac{\partial \mu_0}{\partial c}\right) \in P_1$. Therefore, in view of (2.7), (2.8) the general solution of (3.13) is given by

$$f_1 = v g(\mu_0(t, x, |v|^2), |v|^2) \cdot \left(-\nabla_x \mu_0 - E + E \cdot \frac{\partial \mu_0}{\partial c}\right) + \\ + q(t, x, |v|^2) f_0(1-f_0) \tag{3.14}$$

where q is any function such that $q(t, x, .) \in P_1$. q is still unknown, but (3.14) is enough to compute the flux j_1. We obtain

$$j_1 = j_1(t, x, c) = 2\pi c^2 \sqrt{cg}\,(\mu_0, c)\left(-\nabla_x \mu_0 - E + E \cdot \frac{\partial \mu_0}{\partial c}\right). \tag{3.15}$$

Passing to the limit in (3.5) gives

$$\int_0^{\infty}\left(\partial_t n_0 + \operatorname{div}_x j_1 - 2E \cdot \frac{\partial}{\partial c} j_1\right) p \, dc = 0 \\ \text{for any } p \in p_1 . \tag{3.16}$$

The equations (3.11), (3.15) and (3.16) constitute an energy dependent hydrodynamic model. In order to simplify the formulation we define

$$n(\mu_0, c) = \sum_{k=0}^{\infty} n_0(\mu_0, c+k) \tag{3.17}$$

$$j(t, x, c) = D(\mu_0, c)\left(-\nabla_x \mu_0 - E + E \cdot \frac{\partial \mu_0}{\partial c}\right),$$

$$D(\mu_0, c) = \sum_{k=0}^{\infty} 2\pi (c+k)^2 \sqrt{c+k}\, g(\mu_0, c+k). \tag{3.18}$$

From (3.3) we obtain (formally) :

$$N^{\alpha} \to N = \int_0^1 n(\mu_0(t, x, c), c)\, dc , \tag{3.19}$$

$$J^{\alpha} \to J = \int_0^1 j(t, x, c)\, dc . \tag{3.20}$$

On the other hand, splitting the integral (3.16) in integrals over $(k, k+1)$ and using the change of variable $c \to c + k$ lead to

$$\int_0^1 \left(\partial_t n + \operatorname{div}_x j - 2 E \cdot \frac{\partial}{\partial c} j \right) p\, dc = 0$$

for any $p \in P_1$.

Therefore we obtain

$$\partial_t n + \operatorname{div}_x j - 2 E \frac{\partial}{\partial c} j = 0 \quad t > 0 , \quad x \in IR^3 , \quad c \in (0, 1) \tag{3.21}$$

which, completed by (3.17) and (3.18), constitutes the energy dependent drift diffusion equation.

This equation is subject to periodic boundary conditions

$$n(t, x, 0) = n(t, x, 1) , \quad j(t, x, 0) = j(t, x, 1) . \tag{3.22}$$

Indeed, $n^0(\mu_0, 0) = 0$, $j_1(t, x, 0) = 0$ and (3.22) follows from the definitions (3.17), (3.18).

4. PERTURBATION BY OTHER SCATTERING MECHANISMS

In this Section we study the fluid approximation of the Boltzmann equation when collision processes involve two types of phenomena. The predominant one is always assumed to be the collisions between electrons and phonons which are modelled by the collision operator Q defined by (1.3), (1.5). Other collision mechanisms are modelled by the operator Q_1 defined by (1.3) corresponding to a different cross section σ_1.

We assume that Q_1 is of order α^N compared to Q and that

$$N(Q) \cap N(Q_1) = N_0 . \tag{4.1}$$

As it was already noticed in Section 2, Q_1 could be an operator modelling collisions between electrons and phonons whose energy is irrational (after the scaling). The problem is now to determine the limit of the distribution function f^α which solves

$$\alpha^2 \frac{\partial}{\partial t} f^\alpha + (v \,.\, \nabla_x f^\alpha - E \,.\, \nabla_v f^\alpha) = Q(f^\alpha) + \alpha^N Q_1(f^\alpha)$$

$$t > 0\,, \quad x \in IR^3\,, \quad v \in IR^3\,. \tag{4.2}$$

We restrict ourselves to the cases $N = 1$ or 2. The conservation laws are

$$\left[\partial_t n^\alpha + \mathrm{div}_x j^\alpha - 2\,E \frac{\partial}{\partial c} j^\alpha\right] p\,dc = \alpha^{N-2} \int_{IR^3} Q_1(f^\alpha)\, p(v^2)\, dv\,,$$

$$\text{for any } p \in P_1\,. \tag{4.3}$$

We expand f^α as in (3.6). We find

$$Q(f_0) = 0 \tag{4.4}$$

$$-L_{f_0}(f_1) = -v \,.\, \nabla_x f_0 + E \,.\, \nabla_v f_0 \quad (N = 2) \tag{4.5}$$

$$-Q_1(f_0) - L_{f_0}(f_1) = -v \,.\, \nabla_x f_0 + E \,.\, \nabla_v f_0 \quad (N = 1)\,. \tag{4.6}$$

Therefore f_0 is given by (3.9). In the case $N = 2$, f_1 is determined by (3.14). So if n^0 and j_1 are given by (3.11), (3.12), instead of (3.16) we have

$$\int_0^\infty \left(\partial_t n^0 + \mathrm{div}_x j_i - 2\,E \cdot \frac{\partial}{\partial c} j_1 \right) p\,dc = \int_{IR^3} Q_1(f_0)\, p(|v|^2)\, dv$$

$$\text{for any } p \text{ in } P_1\,. \tag{4.7}$$

Then with the definitions (3.17), (3.18) we obtain

$$\partial_t n + \mathrm{div}_x j - 2\,E \cdot \frac{\partial}{\partial c} j = S\,, \quad t > 0\,, \quad x \in IR^3\,, \quad c \in (0, 1)\,,$$

$$S = \sum_{k=0}^{\infty} \int_{S_2} Q_1(f_0)\,(\sqrt{c+k}\,\Omega)\, d\Omega \tag{4.8}$$

always completed with the boundary condition (3.22). Note that S is a relaxation term. Indeed $S = 0$ implies that

$$\int_{IR^3} Q_1(f_0)\, p(v^2)\, d\sigma = 0 \quad \text{for any} \quad p \in P_1\,. \tag{4.9}$$

But since f_0 is given by (3.9) the function

$$p_0(t, x, .) = a^{v^2} f_0 / (1 - f_0) \in P_1 . \tag{4.10}$$

It follows from Proposition 2.2 that $f_0 \in N(Q_1)$. In view of (4.1) this gives $f_0 \in N_0$. In this case $\mu_0 = \mu_0(t, x)$ does not depend on c. Then we can integrate equation (4.8) with respect to c to obtain the usual drift diffusion model as in [GP].

Now we analyse the case $N = 1$. We multiply (4.6) by p_0 defined by (4.10). In view of (2.9) and since f_0 is an even function we obtain

$$\int Q_1(f_0)\, p_0 \, dv = 0 .$$

As previously, we conclude that $f_0 \in N_0$ and that $\mu_0 = \mu_0(t, x)$ does not depend on c. (4.6) reduces to (4.5) and therefore f_1 is given by (3.14). But (4.3) with $p \equiv 1$ gives

$$\partial_t N^\alpha + \operatorname{div} J^\alpha = 0 .$$

Passing to the limit we obtain

$$\partial_t N + \operatorname{div} J = 0 \tag{4.11}$$

$$N = N(\mu_0(t, x)) = \int_{IR^3} F(\mu_0(t, x), |v|^2)\, dv \tag{4.12}$$

$$J = D(\mu_0(t, x)) (- \nabla_x \mu_0 - E)) \tag{4.13}$$

with

$$D(\mu_0) = \frac{1}{3} \int_{IR^3} g(\mu_0, |v|^2)\, |v|^2 \, dv . \tag{4.14}$$

The drift diffusion model defined by (4.11)-(4.14) is this one which has been already obtained in [GP].

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