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Numerical solutions of the Navier-Stokes equations using wavelet-like incremental unknowns

NUMERICAL SOLUTIONS OF THE NAVIER-STOKES EQUATIONS USING WAVELET-LIKE INCREMENTAL UNKNOWNS (*)

by Theodore TACHIM MEDJO[1]

Abstract — *The multilevel methods developed by M Marion and R Temam [13], for the numerical approximation of dissipative systems is used, along with the wavelet-like incremental unknowns introduced by M Chen and R Temam [3], for the numerical solutions of the two-dimensional Navier-Stokes equations Numerical solutions for the driven cavity flow are obtained with that procedure The spatial splitting of the unknowns improves the stability of the centered difference scheme used hereafter*

Résumé — *Dans cet article, nous utilisons les inconnues incrémentales oscillantes décrites dans [3] pour implémenter les méthodes multi-niveaux proposées dans [13] L'algorithme obtenu est utilisé pour la simulation des équations de Navier-Stokes et le problème de la cavité entraînée pour des nombres de Reynolds élevés*

1. INTRODUCTION

The incremental unknowns method has been introduced in [22] in order to implement the Nonlinear Galerkin Method when finite differences are used for the space discretization. The incremental unknowns are defined using several level of discretization and they correspond to spatial splitting of the unknowns into two (or more) terms which are of different orders of magnitude with respect to the space step. The numerical procedure consists in introducing a simplified approximation for the "small" component, therefore leading to different numerical treatment of the two terms. In particular, the small scale is often obtained as a nonlinear functional of the large scale component. This leads to the Nonlinear Galerkin Method, [13], [14], [22]. These questions have been mainly addressed in the context of spectral Fourier discretization [7], [12], [13]. The case of finite element approximation is considered in [14].

The wavelet-like incremental unknowns (WIU) defined in [3] seem to be particularly well adapted to the implementation of Nonlinear Galerkin Method in the finite differences case. Indeed the WIU enjoy of a L^2-orthogonality which is useful in practice for defining a functional which links the small eddies to the large eddies, [15].

In this article, we use multilevel methods along with the wavelet-like incremental unknowns for the numerical solution of the two dimensional Navier-Stokes equations. Numerical results for the driven cavity flow problem are obtained with that procedure for Reynolds number up to 5000. The spatial splitting of the unknowns improves the stability of the centered difference scheme used hereafter.

The article is divided as follows. In the second and third sections, we recall from [3] the construction of the wavelet-like unknowns. In the fourth section, we applied the procedure for the simulation of the driven cavity flow and we study the behaviour of the incremental component for different Reynolds numbers.

2. THE WAVELET-LIKE INCREMENTAL UNKNOWNS

In this paragraph, we recall from [3] the construction of the wavelet-like incremental unknowns in the two-dimensional case. Let $\Omega = (0, 1)^2$, $N \in \mathbb{N}$, $h = \dfrac{1}{2N+1}$,

$$\Omega_h = \{(ih, jh), 1 \leq i, j \leq 2N\}, \quad \Omega_{2h} = \{(2ih, 2jh), 1 \leq i, j \leq N\}.$$

For $M = (ih, jh)$, we define the function $w_{i,j}$ as follows.

$$w_{i,j} = \begin{cases} 1, & \text{in } [ih, (i+1)h) \times [jh, (j+1)h,) \\ 0, & \text{elsewhere.} \end{cases}$$

We set

$$V_h = \text{span}\{w_{i,j}, 1 \leq i, j \leq 2N\}.$$

For $M \in \Omega_{2h}$, $M = (2ij, 2jh)$, we set

$$\psi_{M,2h} = \begin{cases} 1, & \text{in } [(2i-1)h, (2i+1)h \times ([(2j-1)h, (2j+1)h), \\ 0, & \text{elsewhere.} \end{cases}$$

For $M = (2i-1, 2j)h$, we define the function $\chi_{M,h}$ by

$$\chi_{M,h} = \begin{cases} 1, & \text{in } [(2i-1)h, 2ih) \times [2jh, (2j+1)h), \\ -1, & \text{in } [2ih, (2i+1)h) \times [2jh, (2j+1)h), \\ 0, & \text{elsewhere.} \end{cases}$$

For $M = (2i, 2j-1)h$, we set

$$\chi_{M,h} = \begin{cases} 1, & \text{in } [2ih, (2i+1)h) \times [(2j-1)h, 2jh), \\ -1, & \text{in } [2ih, (2i+1)h) \times [2jh, (2j+1)h), \\ 0, & \text{elsewhere.} \end{cases}$$

For $M = (2i-1, 2j-1)h$, we define $\chi_{M,h}$ by

$$\chi_{M,h} = \begin{cases} 1, & \text{in } [(2i-1)h, 2ih) \times [(2j-1)h, 2jh), \\ -1, & \text{in } [2ih, (2i+1)h) \times [2jh, (2j+1)h), \\ 0, & \text{elsewhere.} \end{cases}$$

Let

$$Y_h = \text{span}\left\{\psi_{M,2h}, M \in \Omega_{2h}\right\} \quad \text{and} \quad Z_h = \text{span}\left\{\chi_{M,h}, M \in \Omega_h \backslash \Omega_{2h}\right\}.$$

From [3], we have the relation

$$V_h = Y_h \oplus Z_h, \quad Y_h \perp Z_h,$$

where the orthogonal relation is taken with respect to the $L^2(\Omega)$ scalar product.

Y_h is the space of coarse grid component and Z_h the space of "small" scale (or incremental) component.

The following figures represent the support of the functions $\psi_{M,2h}, \chi_{A_k,h}$, for $M = (2i, 2j)h$, $A_1 = (2i-1, 2j)h$, $A_2 = (2i, 2j-1)h$, and $A_3 = (2i-1, 2j-1)h$.
For $u \in V_h$,

(2.1) $$u = \sum_{i,j} u_{i,j} w_{i,j};$$

let $y_{2i,2j}, z_{2i,2j-1}, z_{2i-1,2j-1}$, defined by:

(2.2) $$\begin{cases} y_{2i,2j} & = \dfrac{u_{2i,2j} + u_{2i-1,2j} + u_{2i,2j-1} + u_{2i-1,2j-1}}{4} \\ z_{2i-1,2j} & = y_{2i,2j} - u_{2i-1,2j} \\ z_{2i,2j-1} & = y_{2i,2j} - u_{2i,2j-1} \\ z_{2i-1,2j-1} & = y_{2i,2j} - u_{2i-1,2j-1}. \end{cases}$$
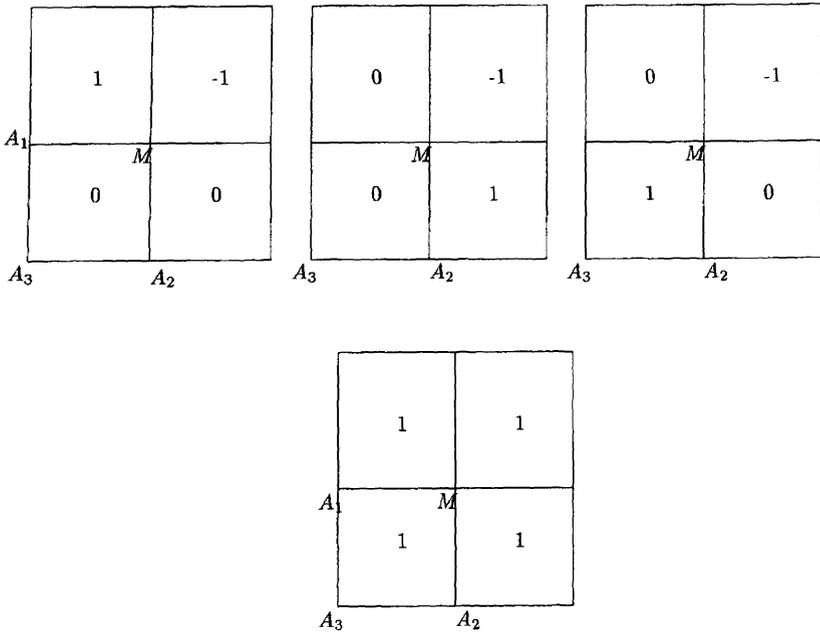
Figure 2.1. — Support of the functions $\chi_{A_1,h}$, $\chi_{A_2,h}$, $\chi_{A_3,h}$ and $\psi_{M,2h}$ respectively.

It is easy to check that (see [3] for more details)

$$u = \sum_{i,j} y_{2i,2j}\, \psi_{2i,2j}$$

(2.3)

$$+ \sum_{i,j} \left(z_{2i-1,2j}\, \chi_{2i-1,2j} + z_{2i,2j-1}\, \chi_{2i,2j-1} + z_{2i-1,2j-1}\, \chi_{2i-1,2j-1} \right).$$

Moreover, $z_{2i-1,2j}$, $z_{2i,2j-1}$, $z_{2i-1,2j-1}$ are of order $O(h)$ and therefore can be considered "small".

3. MULTILEVEL METHODS

In this section, we recall from [3], the multilevel method and the wavelet-like incremental unknowns used hereafter.

As in [3], we consider the class of equations

(3.1) $$\frac{dU}{dt} + AU + R(U) = 0,$$

where $R(U) = B(U) + C(U) + f$, A, B, C, being appropriate operators defined in a Hilbert space H.

Following the framework of [3], [22], we consider a, b and c defined by:

$$B(u, u) = B(u), b(u, v, \omega) = \langle B(u, v), \omega \rangle, c(u, v) = \langle Cu, v \rangle, a(u, v) = \langle Au, v \rangle.$$

Let a_h, b_h, and C_h a family of finite difference approximations of a, b, and C respectively.

We suppose that the following properties hold true:

$$b_h(u_h, v_h, v_h) = 0 \quad \forall u_h, v_h \in V_h,$$

$$|b_h(u_h, v_h, w_h)| \leq c_6 |u_h|^{\frac{1}{2}} \|v_h\|_h^{\frac{1}{2}} \|v_h\|_h |w_h|^{\frac{1}{2}} \|w_h\|_h^{\frac{1}{2}} \quad \forall u_h, v_h, w_h \in V_h,$$

$$|a_h(u_h, v_h)| \leq c_7 \|u_h\|_h \|v_h\|_h \quad \forall u_h, v_h \in V_h,$$

$$|C_h(u_h)| \leq c_8 \|u_h\|_h \quad \forall u_h \in V_h,$$

$$a_h(u_h, u_h) + (C_h(u_h), u_h) \geq \alpha_1 \|u_h\|_h^2 \quad \forall u_h, v_h, w_h \in V_h,$$

where the constants c_i and $\alpha_1 > 0$ are not depending of h, see [3], [22] for more details.

For the space discretization, we are looking for a function u_h from $\mathbb{R}_+$ into V_h satisfying:

$$\frac{d}{dt}(u_h, v_h) + a_h(u_h, v_h) + b_h(u_h, u_h, v_h) + c_h(u_h, v_h) = (f, v_h), \quad \forall v_h \in V_h,$$

where a_h, b_h and c_h are appropriate approximations of a, b and c.

Algorithm

The algorithm that we consider is the following: Find $u_h = y_h + z_h \in V_h$ such that

(3.2)

$$\begin{cases} \frac{d}{dt}(y_h, \bar{y}_h) + a_h(y_h + z_h, \bar{y}_h) + b_h(y_h, y_h, \bar{y}_h) + b_h(y_h, z_h, \bar{y}_h) \\ + b_h(z_h, y_h, \bar{y}_h) + (C_h(y_h + z_h), \bar{y}_h) = (f_h, \bar{y}_h), \quad \forall \bar{y}_h \in Y_h, \\ (y_h(0), \bar{y}_h) = (u_{0h}, \bar{y}_h), \quad \forall \bar{y}_h \in Y_h \\ a_h(z_h, \bar{z}_h) + (C_h z_h, \bar{z}_h) = - a_h(y_h, \bar{z}_h) - b_h(y_h, y_h, \bar{z}_h) + (f_h, \bar{z}_h), \quad \forall \bar{y}_h \in Z_h. \end{cases}$$

By Lax-Milgran theorem and the second equation in (3.2), we have $z_h(t) = \Phi_h(y_h(t))$, see [3], [22] for more details. Thus the system becomes

(3.3)

$$\begin{cases} \dfrac{d}{dt}(y_h, \bar{y}_h) + a_h(y_h + \Phi_h(y_h), \bar{y}_h) + b_h(y_h, y_h, \bar{y}_h) + b_h(y_h, \Phi_h(y_h), \bar{y}_h) \\ + b_h(\Phi_h(y_h), y_h, \bar{y}_h) + (C_h(y_h + \Phi_h(y_h)), \bar{y}_h) = (f_h, \bar{y}_h), \quad \forall \bar{y}_h \in Y_h, \\ (y_h(0), \bar{y}_h) = (u_{0h}, \bar{y}_h), \quad \forall \bar{y}_h \in Y_h, \\ z_h(t) = \Phi_h(y_h(t)). \end{cases}$$

The solution u_h of (3.3) or (3.2) lies on the manifolds $\mathcal{M}_h$ of V_h defined by the equation $z_h = \Phi_h(y_h)$.

The convergence of this algorithm is studied in [3], [22].

4. APPLICATION TO THE NAVIER-STOKES EQUATIONS AND THE SHEAR-DRIVEN CAVITY FLOW

The laminar incompressible flow in a square cavity whose top wall moves with a uniform velocity in its own plane has served over and over again as a model problem for testing and evaluating numerical techniques.

In the following computations, the lid of the cavity moves continuously to the right with a velocity $U = (u, v) = U_b$ at the boundaries (see the figure below).
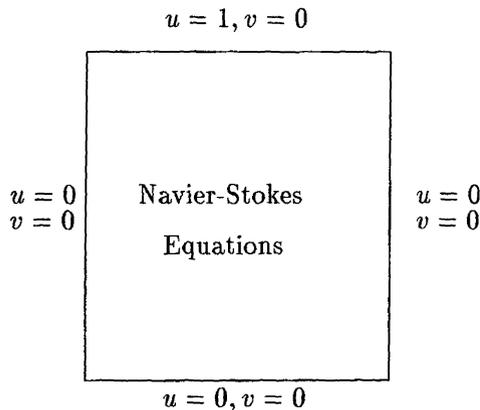
$$u = 1, v = 0$$

$$u = 0 \quad\quad \text{Navier-Stokes} \quad\quad u = 0$$
$$v = 0 \quad\quad \text{Equations} \quad\quad v = 0$$

$$u = 0, v = 0$$

Figure 4.1. — Driven cavity.

Published results are available for this flow problem from a number of sources ([1], [9], [17], [16]) using a variety of solution procedures. The present study aim to obtain these solutions using multilevel methods described above.

Governing differential equations

The two-dimensional flow in the cavity can be represented mathematically in terms of the vorticity and the velocity variables. This formulation, called the vorticity-velocity formulation is represented by the vorticity equation

$$(4.1) \qquad \frac{\partial \omega}{\partial t} - \nu \, \Delta \omega + u \, \frac{\partial \omega}{\partial x} + v \, \frac{\partial \omega}{\partial y} = 0$$

and two Poisson equations for the velocity components u and v

$$(4.2) \qquad \begin{cases} - \Delta u = \dfrac{\partial \omega}{\partial y} \, , \\ - \Delta v = - \dfrac{\partial \omega}{\partial x} \, . \end{cases}$$

The details on the vorcitity-velocity formulation of the Navier-Stokes equations can be found in [6], in which the author gives a closed system of the Navier-Stokes equations and proves its equivalence with the velocity-pressure formulation.

Boundary conditions

The specification of the boundary conditions and their implementation into the numerical scheme represent one of the most important tasks in the development of a finite difference method that is applicable for realistic simulation of unsteady flows. The velocity boundary conditions are given by:

$$(4.3) \qquad \begin{cases} U = (u, v) = (0, 0) \quad \text{on the lines } \; x = 0, y = 0, x = 1 \, , \\ U = (u, v) = (1, 0) \quad \text{on the line } \; y = 1 \, . \end{cases}$$

For the vorticity, since the boundary conditions are not physically known, we used the relation

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

to approximate the boundary conditions. Namely, at time $t = (n + 1) \, \Delta t = t_{n+1}$, we set

$$\omega_{n+1} = \frac{\partial v_n}{\partial x} - \frac{\partial u_n}{\partial y} \, , \quad \text{on} \quad \partial \Omega \, ,$$

where (u_n, v_n) is the approximate value of the velocity at the time $t_n = n\,\Delta t$.

In that relation, the operator $\frac{\partial}{\partial s}$ is approximated by the following finite difference operators of second order

$$\frac{\partial \phi}{\partial s}(x) \simeq \frac{1}{2\,\Delta s}\left[3\,\phi(x) - 4\,\phi(x - \Delta s) + \phi(x - 2\,\Delta s)\right],$$

or

$$\frac{\partial \phi}{\partial s}(x) \simeq \frac{1}{2\,\Delta s}\left[-3\,\phi(x) + 4\,\phi(x + \Delta s) - \phi(x + 2\,\Delta s)\right].$$

Description of the numerical scheme

The finite difference method discussed here stems from the theory of approximate inertial manifolds. By using multilevel methods described above, the approximate solution $(u_h, v_h, \omega_h) = (u, v, \omega)$ is searched into the form

$$u = u_h = u_y + u_z + u_b, \quad v = v_h = v_y + v_z + v_b, \quad \omega = \omega_h = \omega_y + \omega_z + \omega_b,$$

where $(u_y, v_y, \omega_y) \in Y_h \times Y_h \times Y_h$, is called the coarse grid component, $(u_z, v_z, \omega_z) \in Z_h \times Z_h \times Z_h$, is the incremental component, which is supposed to be small in some senses (see [3] for more details) and (u_b, v_b, ω_b) is the boundary conditions described previously.

For the space discretization, the Laplace operator $-\Delta$ is approximated by $-\Delta_h$ defined by:

$$-\Delta_h \phi(x, y) =$$

$$\frac{4\,\phi(x, y) - \phi(x + h, y) - \phi(x - h, y) - \phi(x, y + h) - \phi(x, y - h)}{h^2}.$$

For the nonlinear term of (4.1), the operator $\frac{\partial}{\partial x}$ is approximated by $D_{x,h}$, defined by:

$$D_{x,h}\phi(x) = \frac{1}{2\,h}\left[\phi(x + \Delta x) - \phi(x - \Delta x)\right],$$

with a similar approximation for $\frac{\partial}{\partial y}$.

Let $U_h = (u_h, v_h)$, ϕ, ψ, ω_h defined in Ω_h, we set

$$J_h(U_h, \omega_h) = u_h D_{x,h} \omega_h + v_h D_{y,h} \omega_h,$$

$$j_h(U_h, \omega_{1,h}, \omega_{2,h}) = (J_h(U_h, \omega_{1,h}), \omega_{2,h}),$$

$$a_h(\phi, \psi) = (-\Delta_h \phi, \psi),$$

where $(\,.\,,\,.\,)$ denotes the $L^2(\Omega)$ scalar product.

For the time discretization, we consider the following scheme:

$$\frac{\omega_{n+1} - \omega_n}{\Delta t} - \frac{\nu}{2} \Delta(\omega_{n+1} + \omega_n) + \frac{1}{2}(U_n \cdot \nabla)(\omega_{n+1} + \omega_n) = 0,$$

which follows from the θ-scheme proposed in [16] for the Navier-Stokes equations in the primitive variables (U, p).

Once the vorticity has been computed, the velocity is given by:

$$(4.5) \qquad \begin{cases} -\Delta u_{n+1} = \dfrac{\partial \omega_{n+1}}{\partial y}, \\[2mm] -\Delta v_{n+1} = -\dfrac{\partial \omega_{n+1}}{\partial x}. \end{cases}$$

This scheme has been also used in [18]. For more details on the stability conditions of the θ-scheme, one can see [16].

Let us set

$$U_n = U_y^n + U_z^n + U_b^n, \quad \omega_n = \omega_y^n + \omega_z^n + \omega_b^n,$$

where

$$U_y^n = (u_y^n, v_y^n), \quad U_z^n = (u_z^n, v_z^n), \quad U_b^n = (u_b^n, v_b^n).$$

The multilevel algorithm

Following these notations, the multilevel procedure can be written in the variational form as:

(4.6)

$$\begin{cases} \dfrac{1}{\Delta t}(\omega_y^{n+1} - \omega_y^n, v_h) + \dfrac{1}{2} a_h(\omega_{n+1} + \omega_n, v_h) + \dfrac{1}{2} j_h(U_n, \omega_{n+1} + \omega_n, v_h) = 0, \\ \forall v_h \in Y_h, \\[2mm] \dfrac{1}{\Delta t}(\omega_z^{n+1} - \omega_z^n, v_h) + \dfrac{1}{2} a_h(\omega_{n+1} + \omega_n, v_h) + \dfrac{1}{2} j_h(U_n, \omega_{n+1} + \omega_n, v_h) = 0 \\ \forall v_h \in Z_h, \\[2mm] a_h(u_y^{n+1} + u_z^{n+1} + u_{b,h}, v_h) = (D_{y,h}\omega_{n+1}, v_h), \quad \forall v_h \in Y_h, \\[2mm] a_h(u_y^{n+1} + u_z^{n+1} + u_{b,h}, v_h) = (D_{y,h}\omega_{n+1}, v_h), \quad \forall v_h \in Z_h, \\[2mm] a_h(v_y^{n+1} + v_z^{n+1}, v_h) = (-D_{x,h}\omega_{n+1}, v_h), \quad \forall v_h \in Y_h, \\[2mm] a_h(v_y^{n+1} + v_z^{n+1}, v_h) = (-D_{x,h}\omega_{n+1}, v_h), \quad \forall v_h \in Z_h, \end{cases}$$

At each iteration, the linear system obtained from (4.6) is solved with the GMRES method.

5. NUMERICAL RESULTS

In this paragraph, some typical results are presented, which were obtained with the numerical method described in the previous section.

The test problem considered here is the laminar incompressible flow in a square cavity. The solution are computed for Reynolds numbers up to 5000. We study the evolution of the incremental component with respect to the time. The ratio $\|z\|_{L^2} / \|y\|_{L^2}$ behaves like $O(h)$ for Reynolds numbers up to 2000, and becomes a larger when the Reynolds number increases. This behaviour is probably related to the regularity of the solution with respect to the Reynolds number. In all cases, even as that ratio may not be small enough, we still have $\|z\|_{L^2} < \|y\|_{L^2}$.
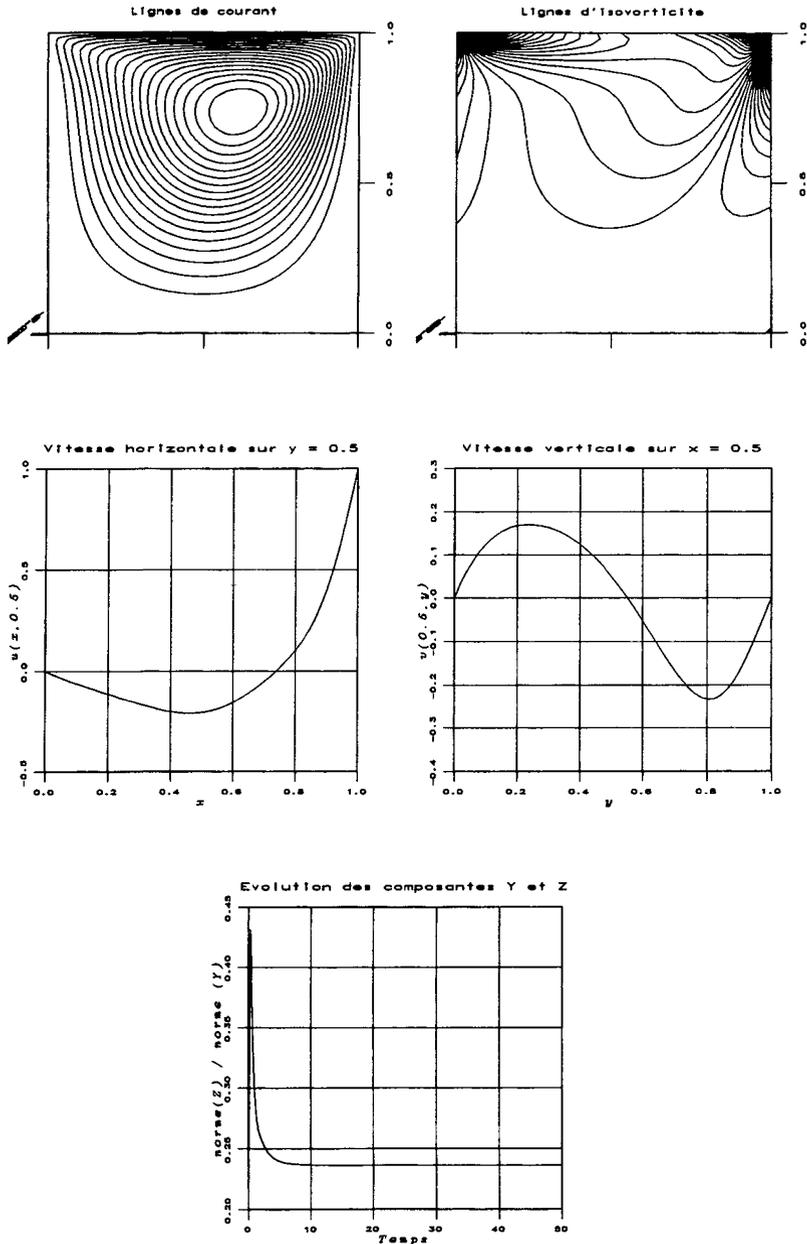
For different values of Reynolds number the following figures represent respectively:

- The streamlines.
- The vorticity contours.
- The u-velocity along the line $y = 0.5$.
- The v-velocity along the line $x = 0.5$.
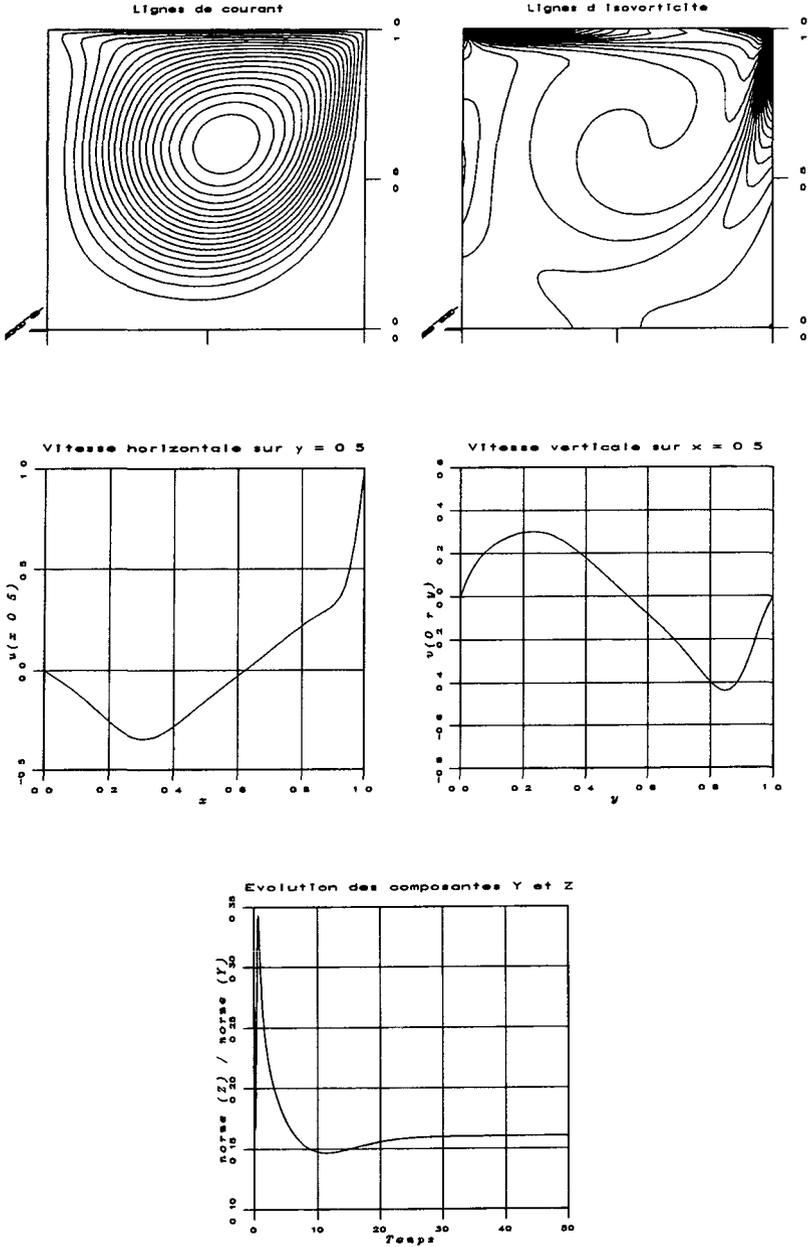- The ratio $\|Z\|_{L^2} / \|Y\|_{L^2} = norm(Z)/norm(Y)$ with respect to the time.

The solutions obtained here are comparable to those obtained in many articles such as [1], [9], [10], [17]. Furthermore, the time step Δt is not too small even if the unconditional stability failed numerically. To insure unconditional stability, one can use a fully implicit scheme rather then the semi-implicit used in this article. The unconvenience of such scheme is the computation time.
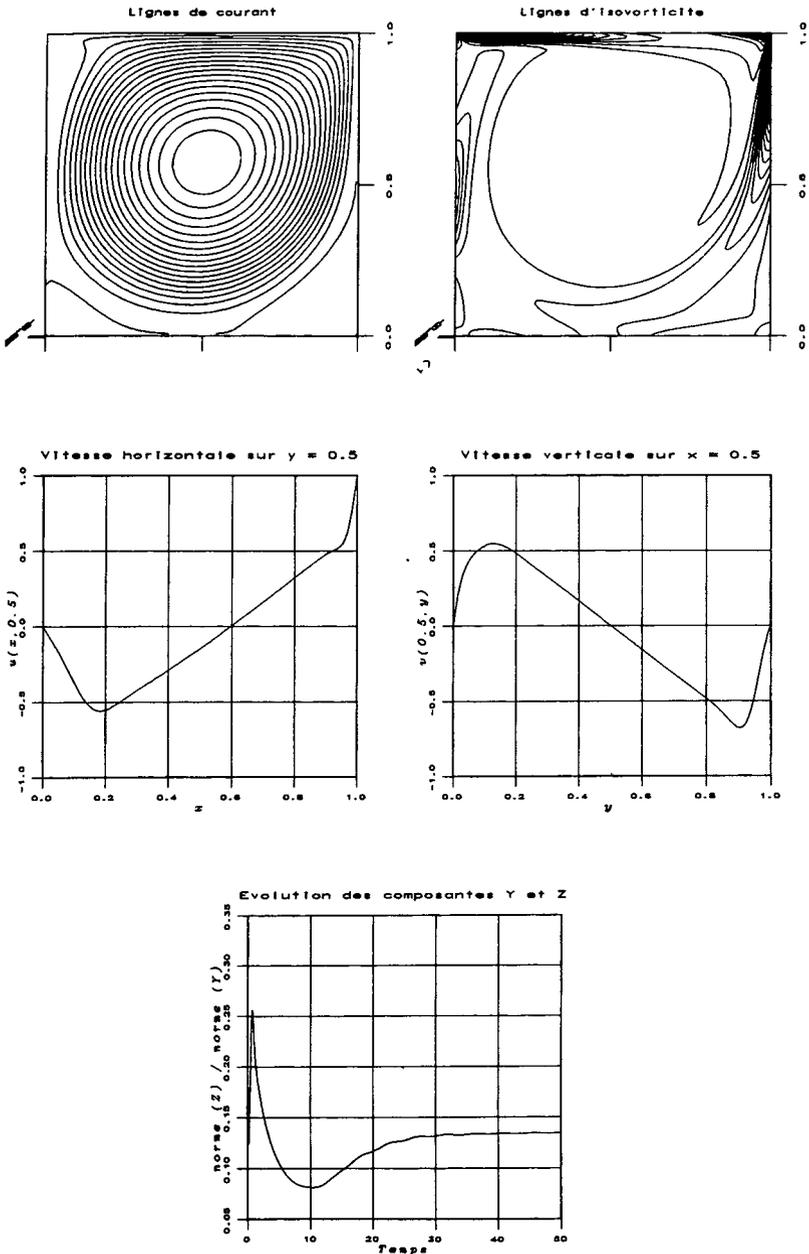
Let us recall the following parameters:

- $d + 1$ is the total number of grid levels.
- N^2 is the number of points in the coarse grid, i.e. the dimension of Y_h.
- $4^d N^2 - N^2$ is the dimension of the incremental component space Z_h.
- $h = \dfrac{1}{2^d N + 1}$ is the grid size.
- Δt is the time step.

Figures 5.1. — Streamlines, Vorticity contours, $u(x, 0.5)$, $v(0.5\,y)$ and the ratio
$\|Z\|_{L^2} / \|Y\|_{L^2} = norm(Z)/norm(Y)$
$N = 25$, $d = 1$, $\Delta t = 0.05$, $Re = 100$
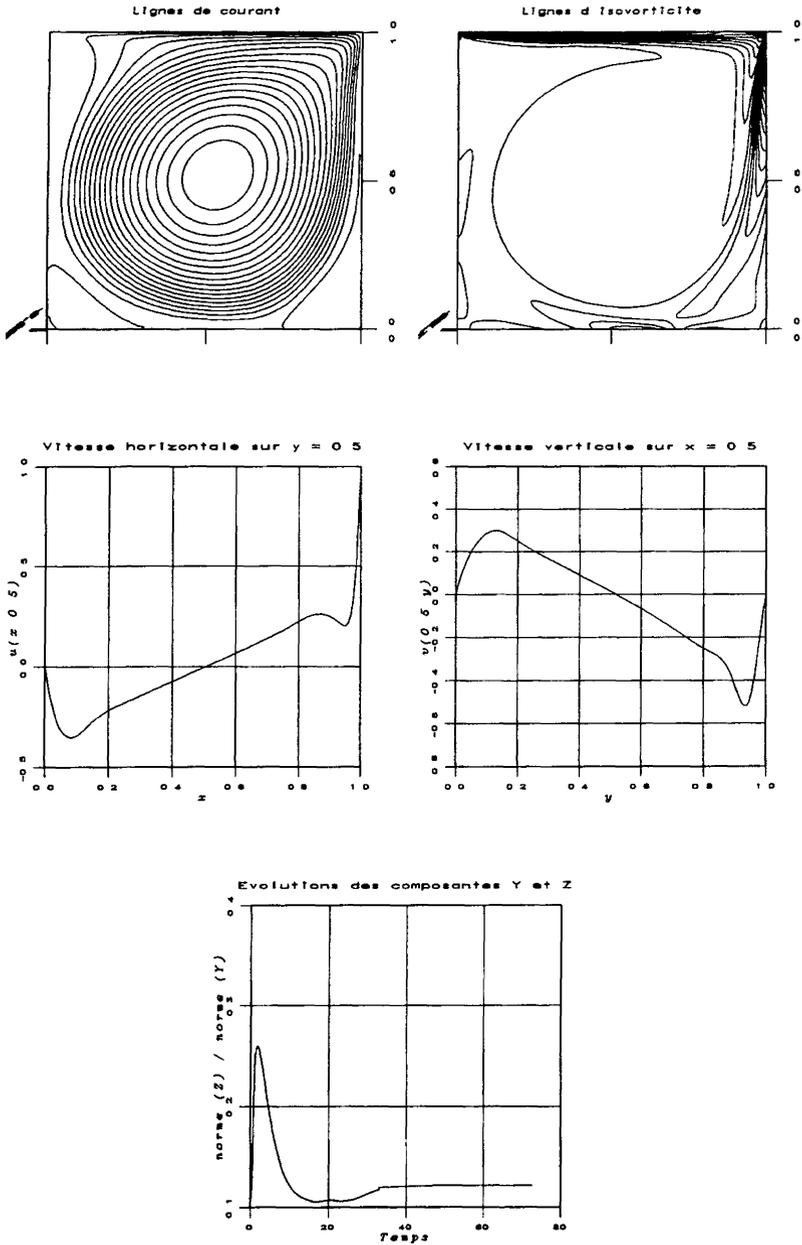
Figures 5.2. — Streamlines, Vorticity contours, $u(x, 0\,5)$, $v(0\,5\,y)$ and the ratio
$\|Z\|_{L^2} / \|Y\|_{L^2} = norm(Z)/norm(Y)$
$N = 32$, $d = 1$, $\Delta t = 0\,05$, $Re = 400$

Lignes de courant

Lignes d'Isovorticite

Vitesse horizontale sur y = 0.5

Vitesse verticale sur x = 0.5

Evolution des composantes Y et Z

Figures 5.3. — Streamlines, Vorticity contours, $u(x, 0.5)$, $v(0.5\,y)$ and the ratio
$\|Z\|_{L^2} / \|Y\|_{L^2} = norm(Z)/norm(Y)$
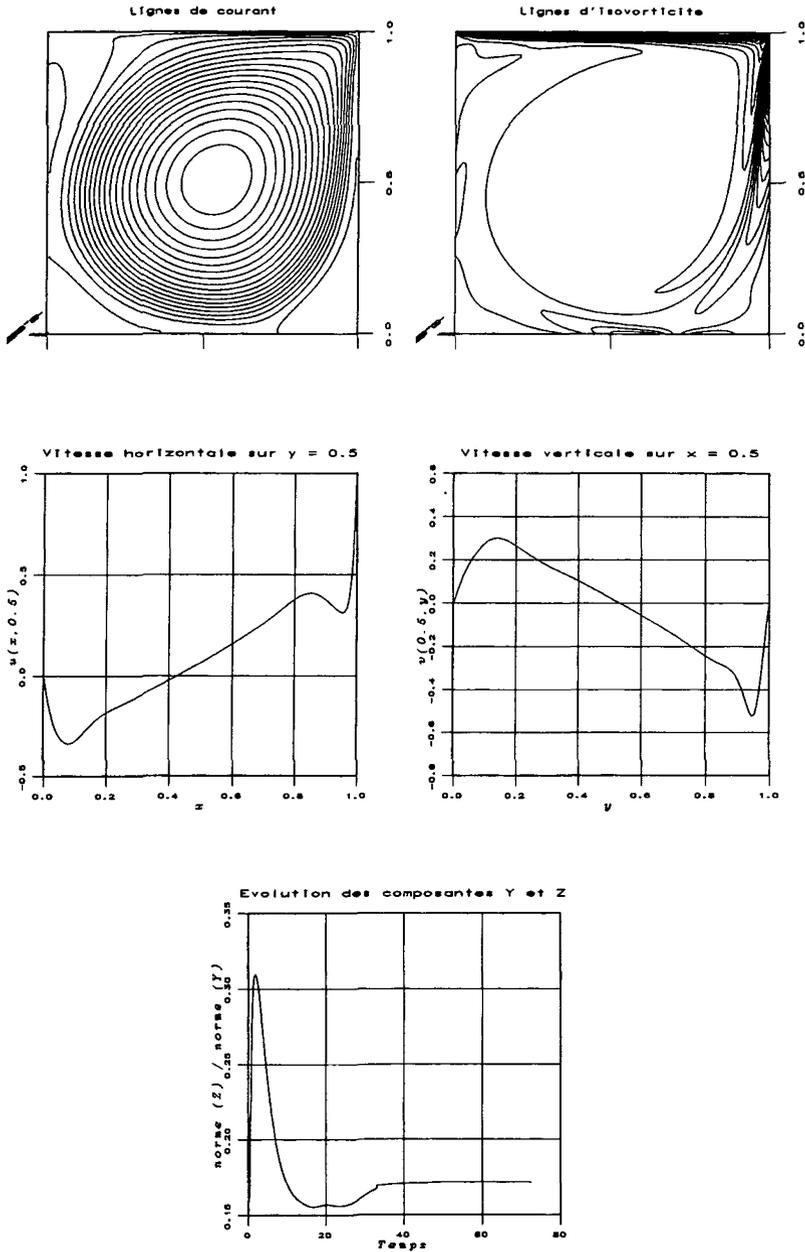$N = 64$, $d = 1$, $\Delta t = 0.05$, $Re = 1\,000$

Figures 5.4. — Streamlines, Vorticity contours, $u(x, 0\,5)$, $v(0\,5\,y)$ and the ratio $\|Z\|_{L^2} / \|Y\|_{L^2} = norm(Z)/norm(Y)$
$N = 64$, $d = 1$, $\Delta t = 0\,05$, $Re = 2\,000$

Lignes de courant

Lignes d'Isovorticite

Vitesse horizontale sur y = 0.5

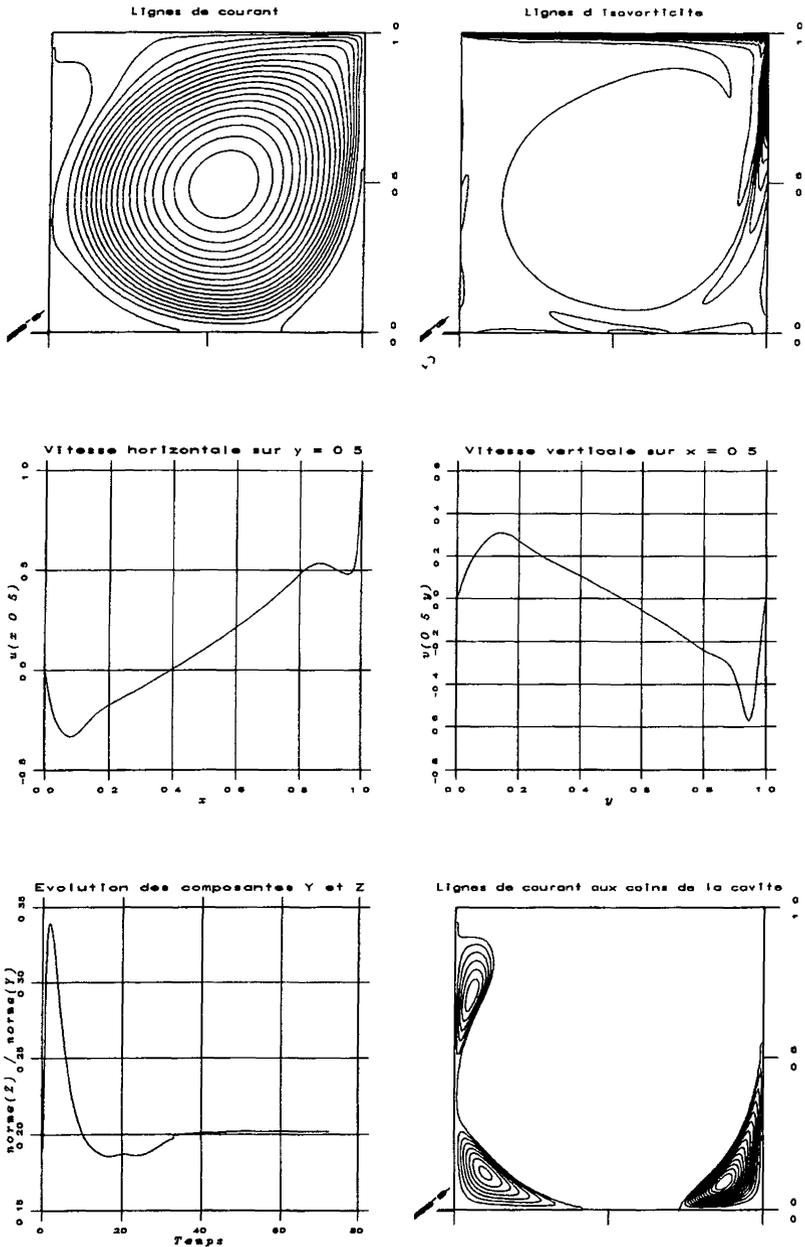Vitesse verticale sur x = 0.5

Evolution des composantes Y et Z

Figures 5.5. — Streamlines, Vorticity contours, $u(x, 0.5)$, $v(0.5 y)$ and the ratio
$\| Z \|_{L^2} / \| Y \|_{L^2} = norm(Z)/norm(Y)$
$N = 64,\quad d = 1,\quad \Delta t = 0.001,\quad Re = 3\,200$

Lignes de courant

Lignes d isovorticite

Vitesse horizontale sur y = 0 5

Vitesse verticale sur x = 0 5

Evolution des composantes Y et Z

Lignes de courant aux coins de la cavite

Figures   5.6. — Streamlines,   Vorticity   contours, $u(x, 0\,5)$, $v(0\,5\,y)$,   the   ratio
$\|Z\|_{L^2} / \|Y\|_{L^2} = norm(Z)/norm(Y)$ **and the streamlines at the corners of the cavity**
$N = 64,\quad d = 1,\quad \Delta t = 0\,001,\quad Re = 5\,000$

6. CONCLUSIONS

A new multilevel method has been used for numerical solutions of the two-dimensional Navier-Stokes equations. Numerical results for the driven cavity flow problem obtained by with that procedure is accurately comparable to published results. The spatial splitting of the unknows improves the stability of the centered difference scheme. This confirm the theoretical results obtained in [22]. We expect that for large number of points, the incremental component (or small scale) will be small enough to be neglected in some terms as described in [22], therefore improving the computation time.

The numerical results presented are performed with two levels of grid points, that is for $d = 1$. Similar computations can be done for $d > 1$ using the multilevel basis proposed in [20]. This generalization is important not only for parallel computers, but also to implement numerical schemes which take into account the "size" of each component. In that case, a different time step can be used for each level. The reader is referred to [7], [12] for more details.

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