ESAIM: M2AN 55 (2021) S877–S878
https://doi.org/10.1051/m2an/2020066

ESAIM: Mathematical Modelling and Numerical Analysis
www.esaim-m2an.org

ERRATUM TO "A C1-P2 FINITE ELEMENT WITHOUT NODAL BASIS"

Shangyou Zhang*

Abstract. A new interpolation operator is defined, which preserves only P2 polynomials locally.

Mathematics Subject Classification. 65N30, 73C35.

Received November 24, 2019. Accepted February 17, 2020.

In [1], a finite element interpolation operator is defined and it is claimed (incorrectly) that the operator preserves the finite element functions (piecewise P_2 polynomials) and that consequently it preserves P_2 polynomials. In this note, we would change the definition of this interpolation operator, from (1) to (5) below, so that it preserves only P_2 polynomials.

In [1], a dual basis ψ_i, corresponding to a finite element nodal basis ϕ_i of $V_{h,0}$, is defined on a 9-square patch M_i as

$$\psi_i = \frac{583\,704}{553\,687}(\phi_{S_1}+\phi_{S_3}+\phi_{S_7}+\phi_{S_9}) - \frac{970\,452}{553\,687}(\phi_{S_2}+\phi_{S_4}+\phi_{S_6}+\phi_{S_8}) + \frac{1\,743\,594}{553\,687}\phi_{S_5}, \tag{1}$$

where S_l, *cf.* Figure 1, is the local indexing of squares around square Q_i ($i = S_5$). Then the local interpolation operator is defined by

$$I_h u = \sum_{i=1}^{n^2} u_i \phi_i, \quad \text{where} \quad u_i = \int_{M_i} \psi_i(\underline{\mathrm{x}}) u(\underline{\mathrm{x}})\, \mathrm{d}\underline{\mathrm{x}}. \tag{2}$$

Lemma 3.2 of [1] mistakenly claims that

$$I_h u = u \quad \text{on} \quad Q_i \quad \text{if} \quad u = \sum_{i=1}^{n^2} u_i \phi_i \quad \text{on} \quad M_i, \tag{3}$$

and that consequently the interpolation preserves P_2 polynomials locally. Because of overlapping, one cannot have (3) but only have

$$I_h u = u \quad \text{on} \quad Q_i \quad \text{if} \quad u = \sum_{l=1}^{9} u_{S_l} \phi_{S_l} \quad \text{on} \quad N_i, \tag{4}$$

where N_i is a patch of 25 squares around Q_i, *cf.* Figure 1. Therefore, I_h does not preserve P_2 polynomials either.

Keywords and phrases. Differentiable finite element, quadratic element, biharmonic equation.

Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, USA.
*Corresponding author: szhang@udel.edu

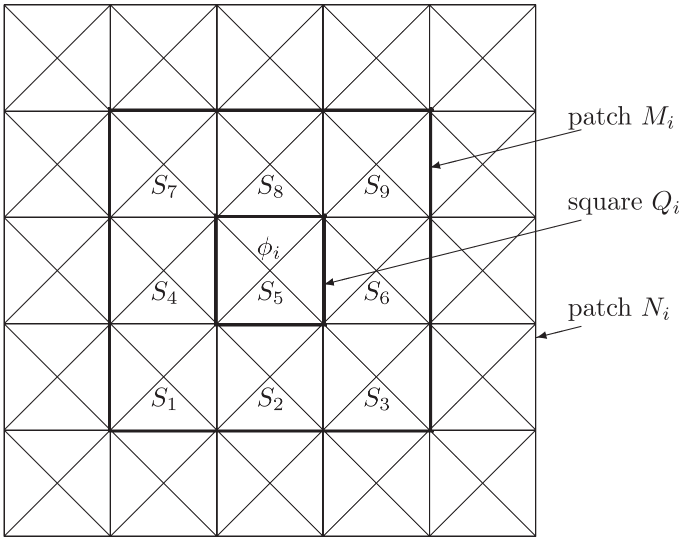

FIGURE 1. Local index mapping around one basis function ϕ_i.

We give up the preservation of all finite element $V_{h,0}$ functions. For preserving only P_2 polynomials locally, the dual basis function ψ_i is simply defined by, replacing (1),

$$\psi_i = 0(\phi_{S_1} + \phi_{S_3} + \phi_{S_7} + \phi_{S_9}) - \frac{45}{386}(\phi_{S_2} + \phi_{S_4} + \phi_{S_6} + \phi_{S_8}) + \frac{493}{772}\phi_{S_5}. \tag{5}$$

Lemma 3.2 of [1] is replaced by the next lemma.

Lemma 1. *If $u \in P_2(N_i)$, cf. Figure 1, then*

$$I_h u = u \quad \text{on} \quad Q_i,$$

where I_h is defined in (2) with ψ_i defined in (5).

Proof. Let the lower-left corner of square Q_{S_1} be $(0,0)$, *cf.* Figure 1, and the grid size $h = 1$.

$$\int_{M_i} \psi_i(x,y)u(x,y)\,\mathrm{d}x\,\mathrm{d}y = \begin{cases} 1/2 & \text{if } u = 1, \\ 3/4 & \text{if } u = x, \quad \text{or } y, \\ 1 & \text{if } u = x^2, \quad \text{or } y^2, \\ 9/8 & \text{if } u = xy. \end{cases}$$

Combining 9 terms, with the definition of ϕ_{S_l}, we get $I_h x^k y^l = x^k y^l$ on Q_i for $k + l \le 2$. □

Remark 1. The rest analysis and results remain same in [1], after the above correction.

REFERENCES

[1] S. Zhang, A C1-P2 finite element without nodal basis. *M2AN* **42** (2008) 175–192.