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ERRATUM TO: "SPARSE-GRID POLYNOMIAL INTERPOLATION APPROXIMATION AND INTEGRATION FOR PARAMETRIC AND STOCHASTIC ELLIPTIC PDES WITH LOGNORMAL INPUTS" [ERRATUM TO ESAIM: M2AN 55 (2021) 1163-1198]

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We correct some errors in [1] which are mostly related to the integration results. For more details, see [2]. For convenience, we keep the same numbering as in [1] of equations and of the theorems and corollaries to be corrected.

1. In the proof Lemma 3.5 in Section 3 from [1], the paragraph lying between the begining of the page ([1], 1175) and the end of this proof is corrected as follows.

For the norm $\|v_\xi - \mathcal{S}_{G(\xi)}v\|_{\mathcal{L}_2(X^1)}$, with $\alpha^* := \alpha + 1/2$ and $N = N(\xi, \boldsymbol{s}) := 2^{\left\lfloor \log_2\left(\sigma_{2;\boldsymbol{s}}^{-1/\alpha^*}\xi^{\vartheta/\alpha^*}\right)\right\rfloor}$ we have

$$\begin{aligned}
\left\|v_\xi - \mathcal{S}_{G(\xi)}v\right\|_{\mathcal{L}_2(X^1)} &\le \sum_{\sigma_{1;\boldsymbol{s}}^{q_1} \le \xi} \left\| v_{\boldsymbol{s}} - \sum_{2^k \le \sigma_{2;\boldsymbol{s}}^{-1/\alpha^*}\xi^{\vartheta/\alpha^*}} \delta_k(v_{\boldsymbol{s}}) \right\|_{X^1} \|H_{\boldsymbol{s}}\|_{L_2(\mathbb{R}^\infty,\gamma)} \\
&= C \sum_{\sigma_{1;\boldsymbol{s}}^{q_1} \le \xi} \|v_{\boldsymbol{s}} - P_N(v_{\boldsymbol{s}})\|_{X^1} \le C \sum_{\sigma_{1;\boldsymbol{s}}^{q_1} \le \xi} N^{-\alpha} \|v_{\boldsymbol{s}}\|_{X^2} \\
&\le C \sum_{\sigma_{1;\boldsymbol{s}}^{q_1} \le \xi} \left(\sigma_{2;\boldsymbol{s}}^{-1/\alpha^*}\xi^{\vartheta/\alpha^*}\right)^{-\alpha} \|v_{\boldsymbol{s}}\|_{X^2} \le C\, \xi^{-\vartheta\alpha/\alpha^*} \sum_{\sigma_{1;\boldsymbol{s}}^{q_1} \le \xi} \sigma_{2;\boldsymbol{s}}^{\alpha/\alpha^*} \|v_{\boldsymbol{s}}\|_{X^2} \\
&\le C\, \xi^{-\vartheta\alpha/\alpha^*} \left(\sum_{\sigma_{1;\boldsymbol{s}}^{q_1} \le \xi} (\sigma_{2;\boldsymbol{s}} \|v_{\boldsymbol{s}}\|_{X^2})^2 \right)^{1/2} \left(\sum_{\sigma_{1;\boldsymbol{s}}^{q_1} \le \xi} \sigma_{2;\boldsymbol{s}}^{2(\alpha/\alpha^* - 1)} \right)^{1/2} \\
&\le C\, \xi^{-\vartheta\alpha/\alpha^*} \left(\sum_{\sigma_{1;\boldsymbol{s}}^{q_1} \le \xi} \sigma_{2;\boldsymbol{s}}^{-1/\alpha^*} \right)^{1/2}.
\end{aligned}$$

With $q := q_2\alpha^* > 1$ and $1/q + 1/q' = 1$, by the Hölder inequality we obtain

$$\sum_{\sigma_{1;\boldsymbol{s}}^{q_1} \le \xi} \sigma_{2;\boldsymbol{s}}^{-1/\alpha^*} \le \left(\sum_{\sigma_{1;\boldsymbol{s}}^{q_1} \le \xi} \sigma_{2;\boldsymbol{s}}^{-q_2} \right)^{1/q} \left(\sum_{\sigma_{1;\boldsymbol{s}}^{q_1} \le \xi} 1 \right)^{1/q'} \le C \left(\sum_{\sigma_{1;\boldsymbol{s}}^{q_1} \le \xi} \sigma_{1;\boldsymbol{s}}^{-q_1} \xi \right)^{1/q'} \le C\, \xi^{-1/q'}.$$

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Summing up, we find

$$\|v_\xi - \mathcal{S}_{G(\xi)} v\|_{\mathcal{L}_2(X^1)} \le C\,\xi^{-\vartheta\alpha/\alpha^* + 1/2q'} = C\,\xi^{-(1/q_1 - 1/2)}$$

due to the equality $-\vartheta\alpha/\alpha^* + 1/2q' = -(1/q_1 - 1/2)$. This, equations (3.8) and (3.9) prove the lemma for the case $\alpha > 1/q_2 - 1/2$. □

2. Theorem 3.10 and Corollary 3.12 in [1] are incorrect. Therefore, the results on integration based on them, in particular, Theorems 4.1, 5.10 and Corollaries 4.2, 5.11 in [1] are also incorrect. Below we give an illumination of this incorrectness and corrections of these results and their proofs which are just slight modifications of them.

2.1. Let us analyze a main error in the proof of Theorem 3.10 from [1], for example, for the case $\alpha \le 1/q_2 - 1/2$ which leads to its incorrectness. This very short proof ([1], page 1183) says that it is similar to the proof of Theorem 3.8 from [1] with some modifications. For example, all the indices sets are taken from the sets $\mathbb{F}_{\rm ev}$ and $\mathbb{N}_0 \times \mathbb{F}_{\rm ev}$ instead of $\mathbb{F}$ and $\mathbb{N}_0 \times \mathbb{F}$. Notice that in the proof of Theorem 3.8 from [1], we used the crucial equality $I_\Lambda H_{\boldsymbol{s}} = H_{\boldsymbol{s}}$ for every $\boldsymbol{s} \in \Lambda$ which holds for the interpolation operator I_Λ defined in [1], page 1177, if Λ is a downward closed set in $\mathbb{F}$. More precisely, this equality then is applied to the downward closed sets Λ_k defined in [1], page 1178. For details, see [1], pages 1178–1180. However, the sets

$$\Lambda_{{\rm ev},k} := \{\boldsymbol{s} \in \mathbb{F}_{\rm ev} : (k, \boldsymbol{s}) \in G_{\rm ev}(\xi)\} = \left\{\boldsymbol{s} \in \mathbb{F}_{\rm ev} : \sigma_{2,\boldsymbol{s}}^{q_2} \le 2^{-k}\xi\right\},$$

to be used in a similar way in the proof of Theorem 3.10 from [1], are *not downward closed sets in* $\mathbb{F}$, where $G_{\rm ev}(\xi)$ is defined in (3.11) of [1]. Hence the proof of Theorem 3.10 from [1] is faulted. There is the same error in the proof of Corollary 3.12 from [1].

2.2. To have a correct formulation and proof of Theorem 3.10 and Corollary 3.12 in Section 3 from [1] we need some modifications of the definitions of I_Λ and $\mathcal{I}_G$ for finite sets $\Lambda \subset \mathbb{F}_{\rm ev}$ and $G \subset \mathbb{N}_0 \times \mathbb{F}_{\rm ev}$, and an extension of concept of downward closed set in $\mathbb{F}_{\rm ev}$. Recall that the definitions of I_Λ and $\mathcal{I}_G$ for finite sets $\Lambda \subset \mathbb{F}$ and $G \subset \mathbb{N}_0 \times \mathbb{F}$ are given in [1], page 1177.

For a given sequence $(Y_m)_{m=0}^\infty$, we define the univariate operator $\Delta_m^{\rm I*}$ for even $m \in \mathbb{N}_0$ by

$$\Delta_m^{\rm I*} := I_m - I_{m-2},$$

with the convention $I_{-2} = 0$.

The operators $\Delta_{\boldsymbol{s}}^{\rm I*}$ for $\boldsymbol{s} \in \mathbb{F}_{\rm ev}$, I_Λ^* for a finite set $\Lambda \subset \mathbb{F}_{\rm ev}$ and $\mathcal{I}_G^*$ for a finite set $G \subset \mathbb{N}_0 \times \mathbb{F}_{\rm ev}$, are defined in similar way as $\Delta_{\boldsymbol{s}}^{\rm I}$, I_Λ and $\mathcal{I}_G$ in Section 3 from [1] by replacing $\Delta_{s_j}^{\rm I}$ with $\Delta_{s_j}^{\rm I*}$, $j \in \mathbb{N}$.

A set Λ is called downward closed in $\mathbb{F}_{\rm ev}$ if $\Lambda \subset \mathbb{F}_{\rm ev}$ and the inclusion $\boldsymbol{s} \in \Lambda$ yields the inclusion $\boldsymbol{s}' \in \Lambda$ for every $\boldsymbol{s}' \in \mathbb{F}_{\rm ev}$ such that $\boldsymbol{s}' \le \boldsymbol{s}$. A sequence $(\sigma_{\boldsymbol{s}})_{\boldsymbol{s} \in \mathbb{F}_{\rm ev}}$ is called increasing in $\mathbb{F}_{\rm ev}$ if $\sigma_{\boldsymbol{s}'} \le \sigma_{\boldsymbol{s}}$ for every $\boldsymbol{s}, \boldsymbol{s}' \in \mathbb{F}_{\rm ev}$ such that $\boldsymbol{s}' \le \boldsymbol{s}$. Put $R_{{\rm ev};\boldsymbol{s}} := \{\boldsymbol{s}' \in \mathbb{F}_{\rm ev} : \boldsymbol{s}' \le \boldsymbol{s}\}$. Here, recall that the inequality $\boldsymbol{s}' \le \boldsymbol{s}$ means $s_j' \le s_j$ for every $j \in \mathbb{N}$.

One can verify that $I_\Lambda^* H_{\boldsymbol{s}} = H_{\boldsymbol{s}}$ for every $\boldsymbol{s} \in \Lambda$ if Λ is a downward closed set in $\mathbb{F}_{\rm ev}$, and that the sets $\Lambda_{{\rm ev},k}$ defined in 2.1 are indeed downward closed in $\mathbb{F}_{\rm ev}$. These properties are actually used in the proofs of the corrections of Theorem 3.10 and Corollary 3.12 from [1] below.

2.3. Theorem 3.10 and Corollary 3.12 in Section 3 from [1] and their proofs are corrected by replacing the interpolation operators $\mathcal{I}_{G_{\rm ev}(\xi_n)}$ and $I_{\Lambda_{\rm ev}(\xi_n)}$ with $\mathcal{I}^*_{G_{\rm ev}(\xi_n)}$ and $I^*_{\Lambda_{\rm ev}(\xi_n)}$, respectively. They are as follows.

Theorem 3.10. *Let* $0 < p \le 2$. *Let Assumption 2.1 hold for Hilbert spaces* X^1 *and* X^2. *Let* $v \in \mathcal{L}_2^{\mathcal{E}}(X^2)$ *be represented by the series* (3.10). *Assume that* $(Y_m)_{m\in\mathbb{N}_0}$ *is a sequence satisfying the condition* (3.15) *for some positive numbers* τ *and* C. *Assume that for* $r = 1, 2$ *there exist increasing sequences* $(\sigma_{r;\boldsymbol{s}})_{\boldsymbol{s}\in\mathbb{F}_{\rm ev}}$ *of numbers strictly larger than* 1 *such that*

$$\sum_{\boldsymbol{s}\in\mathbb{F}_{\rm ev}} (\sigma_{r;\boldsymbol{s}} \|v_{\boldsymbol{s}}\|_{X^r})^2 < \infty$$

and $(p_{\boldsymbol{s}}(2\theta,\lambda)\sigma_{r;\boldsymbol{s}}^{-1})_{\boldsymbol{s}\in\mathbb{F}_{\rm ev}} \in \ell_{q_r}(\mathbb{F}_{\rm ev})$ *for some* $0 < q_1 \le q_2 < \infty$ *with* $q_1 < 2$*, where* θ *and* λ *are as in* (3.18)*. For* $\xi > 0$*, let* $G_{\rm ev}(\xi)$ *be the set defined as in* (3.11)*. Then for each* $n \in \mathbb{N}$ *there exists a number* ξ_n *such that for the operator* $\mathcal{I}^*_{G_{\rm ev}(\xi_n)} : \mathcal{L}_2^{\mathcal{E}}(X^2) \to \mathcal{V}(G(\xi_n))$*, we have that* $\dim \mathcal{V}(G_{\rm ev}(\xi_n)) \le n$ *and*

$$\left\| v - \mathcal{I}^*_{G_{\rm ev}(\xi_n)} v \right\|_{\mathcal{L}_p(X^1)} \le C n^{-\min(\alpha,\beta)}. \tag{3.37}$$

The rate α *corresponds to the approximation of a single function in* X^2 *as given by* (2.3) *The rate* β *is given by* (3.20)*. The constant* C *in* (3.37) *is independent of* v *and* n*.*

Proof. The proof of this theorem is similar to the proof of Theorem 3.8 with some modifications. For example, the sets $\mathbb{F}$ and $\mathbb{N}_0 \times \mathbb{F}$ are replaced by $\mathbb{F}_{\rm ev}$ and $\mathbb{N}_0 \times \mathbb{F}_{\rm ev}$, the sets $G(\xi)$ by $G_{\rm ev}(\xi)$ and the sets $R_{\boldsymbol{s}}$ by $R_{{\rm ev};\boldsymbol{s}}$; the operators $\mathcal{I}_{G_{\rm ev}(\xi)}$ are replaced by $\mathcal{I}^*_{G_{\rm ev}(\xi)}$; the sets $\Lambda \subset \mathbb{F}_{\rm ev}$ and $\Lambda_{{\rm ev},k} \subset \mathbb{F}_{\rm ev}$ are downward closed in $\mathbb{F}_{\rm ev}$; the equality $I_\Lambda H_{\boldsymbol{s}} = H_{\boldsymbol{s}}$ for every $\boldsymbol{s} \in \Lambda$ and downward closed set Λ in $\mathbb{F}$, is replaced by the equality $I^*_\Lambda H_{\boldsymbol{s}} = H_{\boldsymbol{s}}$ for every $\boldsymbol{s} \in \Lambda$ and downward closed set Λ in $\mathbb{F}_{\rm ev}$; estimates similar to (3.24) and (3.32) are given by Lemma 3.6 instead of Lemma 3.5. □

In a similar way we prove the following

Corollary 3.12. *Let* $v \in \mathcal{L}_2^{\mathcal{E}}(X)$ *be represented by the series* (3.10) *for a Hilbert space* X*. Assume that* $(Y_m)_{m\in\mathbb{N}_0}$ *is a sequence satisfying the condition* (3.15) *for some positive numbers* τ *and* C*. Assume that there exists an increasing sequence* $(\sigma_{\boldsymbol{s}})_{\boldsymbol{s}\in\mathbb{F}_{\rm ev}}$ *of numbers strictly larger than* 1 *such that*

$$\sum_{\boldsymbol{s}\in\mathbb{F}_{\rm ev}} (\sigma_{\boldsymbol{s}} \|v_{\boldsymbol{s}}\|_X)^2 < \infty$$

and $(p_{\boldsymbol{s}}(2\theta, \max(2,\lambda))\sigma_{\boldsymbol{s}}^{-1})_{\boldsymbol{s}\in\mathbb{F}_{\rm ev}} \in \ell_q(\mathbb{F}_{\rm ev})$ *for some* $0 < q < 2$*, where* θ *and* λ *are as in* (3.18)*. For* $\xi > 0$*, define*

$$\Lambda_{\rm ev}(\xi) := \{\boldsymbol{s} \in \mathbb{F}_{\rm ev} : \sigma_{\boldsymbol{s}}^q \le \xi\}. \tag{3.41}$$

Then for each $m \in \mathbb{N}$ *there exists a number* ξ_n *such that* $|\Gamma(\Lambda_{\rm ev}(\xi_n))| \le n$ *and*

$$\left\| v - I^*_{\Lambda_{\rm ev}(\xi_n)} v \right\|_{\mathcal{L}_p(X)} \le C n^{-(1/q-1/2)}. \tag{3.42}$$

The constant C *in* (3.42) *is independent of* v *and* n*.*

2.4. The equality $y_{m;m-k} = y_{m;k}$ in the line ([1], page 1185, line 7) is corrected as $y_{m;m-k} = -y_{m;k}$.

2.5. The definitions of integration operators in Section 4 from [1] are corrected as follows.

For a given sequence $(Y_m)_{m=0}^\infty$, we define the univariate operator $\Delta_m^{\rm Q}$ for even $m \in \mathbb{N}_0$ by

$$\Delta_m^{\rm Q} := Q_m - Q_{m-2},$$

with the convention $Q_{-2} := 0$.

For a function $v \in \mathcal{L}_2^{\mathcal{E}}(X)$, we introduce the operator $\Delta_{\boldsymbol{s}}^{\rm Q}$ defined for $\boldsymbol{s} \in \mathbb{F}_{\rm ev}$ by

$$\Delta_{\boldsymbol{s}}^{\rm Q}(v) := \bigotimes_{j\in\mathbb{N}} \Delta_{s_j}^{\rm Q}(v),$$

where the univariate operator $\Delta_{s_j}^{\rm Q}$ is applied to the univariate function v by considering v as a function of variable y_i with the other variables held fixed. For a finite set $\Lambda \subset \mathbb{F}_{\rm ev}$, we introduce the quadrature operator Q_Λ which is generated by the interpolation operator I^*_Λ as follows

$$Q_\Lambda v := \sum_{\boldsymbol{s}\in\Lambda} \Delta_{\boldsymbol{s}}^{\rm Q}(v) = \int_{\mathbb{R}^\infty} I^*_\Lambda v(\boldsymbol{y})\, {\rm d}\gamma(\boldsymbol{y}).$$

Further, if $\phi \in X'$ is a bounded linear functional on X, denote by $\langle \phi, v\rangle$ the value of ϕ in v. For a finite set $\Lambda \subset \mathbb{F}_{\rm ev}$, the quadrature formula $Q_\Lambda v$ generates the quadrature formula $Q_\Lambda\langle \phi, v\rangle$ for integration of $\langle \phi, v\rangle$ by

$$Q_\Lambda\langle \phi, v\rangle := \langle \phi, Q_\Lambda\rangle = \int_{\mathbb{R}^\infty} \langle \phi, I_\Lambda^* v(\boldsymbol{y})\rangle \,\mathrm{d}\gamma(\boldsymbol{y}).$$

Let Assumption 2.1 hold for Hilbert spaces X^1 and X^2, and $v \in \mathcal{L}_2^{\mathcal{E}}(X^2)$. For a finite set $G \subset \mathbb{N}_0 \times \mathbb{F}_{\rm ev}$, we introduce the quadrature operator $\mathcal{Q}_G$ which is generated by the interpolation operator $\mathcal{I}_G^* : \mathcal{L}_2^{\mathcal{E}}(X^2) \to \mathcal{V}(G)$, and which is defined for v by

$$\mathcal{Q}_G v := \sum_{(k,\boldsymbol{s})\in G} \delta_k \Delta_{\boldsymbol{s}}^{\rm Q}(v) = \int_{\mathbb{R}^\infty} \mathcal{I}_G^* v(\boldsymbol{y})\,\mathrm{d}\gamma(\boldsymbol{y}). \tag{4.1}$$

Further, if $\phi \in (X^1)'$ is a bounded linear functional on X^1, for a finite set $G \subset \mathbb{N}_0 \times \mathbb{F}_{\rm ev}$, the quadrature formula $\mathcal{Q}_G v$ generates the quadrature formula $\mathcal{Q}_G\langle \phi, v\rangle$ for integration of $\langle \phi, v\rangle$ by

$$\mathcal{Q}_G\langle \phi, v\rangle := \langle \phi, \mathcal{Q}_G v\rangle = \int_{\mathbb{R}^\infty} \langle \phi, \mathcal{I}_G^* v(\boldsymbol{y})\rangle \,\mathrm{d}\gamma(\boldsymbol{y}).$$

2.6. Theorems 4.1 in Section 4 of [1] and its proof are corrected by replacing the interpolation operators $\mathcal{I}_{G_{\rm ev}(\xi_n)}$ with $\mathcal{I}^*_{G_{\rm ev}(\xi_n)}$. They are as follows.

Theorem 4.1. *Under the hypothesis of Theorem 3.8, assume additionally that the sequences Y_m, $m \in \mathbb{N}_0$, are symmetric. For $\xi > 0$, let $G_{\rm ev}(\xi)$ be the set defined as in* (3.11)*. Then for the quadrature operator $\mathcal{Q}_{G_{\rm ev}(\xi)}$ generated by the interpolation operator $\mathcal{I}^*_{G_{\rm ev}(\xi)} : \mathcal{L}_2^{\mathcal{E}}(X^2) \to \mathcal{V}(G_{\rm ev}(\xi))$, we have the following*

(i) *For each $n \in \mathbb{N}$ there exists a number ξ_n such that* $\dim \mathcal{V}(G_{\rm ev}(\xi_n)) \le n$ *and*

$$\left\| \int_{\mathbb{R}^\infty} v(\boldsymbol{y})\,\mathrm{d}\gamma(\boldsymbol{y}) - \mathcal{Q}_{G_{\rm ev}(\xi_n)} v \right\|_{X^1} \le C n^{-\min(\alpha,\beta)}. \tag{4.4}$$

(ii) *Let $\phi \in (X^1)'$ be a bounded linear functional on X^1. Then for each $n \in \mathbb{N}$ there exists a number ξ_n such that* $\dim \mathcal{V}(G_{\rm ev}(\xi_n)) \le n$ *and*

$$\left| \int_{\mathbb{R}^\infty} \langle \phi, v(\boldsymbol{y})\rangle \,\mathrm{d}\gamma(\boldsymbol{y}) - \mathcal{Q}_{G_{\rm ev}(\xi_n)}\langle \phi, v\rangle \right| \le C n^{-\min(\alpha,\beta)}. \tag{4.5}$$

The rate α corresponds to the approximation of a single function in X^2 as given by (2.3)*. The rate β is given by* (3.20)*. The constants C in* (4.4) *and* (4.5) *are independent of v and n.*

Proof. For a given $n \in \mathbb{N}$, we approximate the integral $\int_{\mathbb{R}^\infty} v(\boldsymbol{y})\,\mathrm{d}\gamma(\boldsymbol{y})$ by $\mathcal{Q}_{G_{\rm ev}(\xi_n)}$ where ξ_n is as in Theorem 3.10. By Lemmata 3.3 and 3.4 the series (2.5) and (3.4) converge absolutely, and therefore, unconditionally in the Hilbert space $\mathcal{L}_2(X^1)$ to v. Hence, by (4.3) we derive that $\mathcal{Q}_{G_{\rm ev}(\xi_n)} v = \mathcal{Q}_{G_{\rm ev}(\xi_n)} v_{\rm ev}$. Due to (4.1) and (4.2) there holds the equality

$$\int_{\mathbb{R}^\infty} v(\boldsymbol{y})\,\mathrm{d}\gamma(\boldsymbol{y}) - \mathcal{Q}_{G_{\rm ev}(\xi_n)} v = \int_{\mathbb{R}^\infty} \Big(v_{\rm ev}(\boldsymbol{y}) - \mathcal{I}^*_{G_{\rm ev}(\xi_n)} v_{\rm ev}(\boldsymbol{y}) \Big)\,\mathrm{d}\gamma(\boldsymbol{y}). \tag{4.6}$$

Hence, applying (3.37) in Theorem 3.10 for $p = 1$, we obtain (i):

$$\left\| \int_{\mathbb{R}^\infty} v(\boldsymbol{y})\,\mathrm{d}\gamma(\boldsymbol{y}) - \mathcal{Q}_{G_{\rm ev}(\xi_n)} v \right\|_{X^1} \le \left\| v_{\rm ev} - \mathcal{I}^*_{G_{\rm ev}(\xi_n)} v_{\rm ev} \right\|_{\mathcal{L}_1(X^1)} \le C n^{-\min(\alpha,\beta)}.$$

For a given $n \in \mathbb{N}$, we approximate the integral $\int_{\mathbb{R}^\infty} \langle \phi, v(\boldsymbol{y}) \rangle \, \mathrm{d}\gamma(\boldsymbol{y})$ by $\mathcal{Q}_{G_{\mathrm{ev}}(\xi_n)} \langle \phi, v \rangle$ where ξ_n is as in Corollary 3.12. Similarly to (4.6), there holds the equality

$$\int_{\mathbb{R}^\infty} \langle \phi, v(\boldsymbol{y}) \rangle \, \mathrm{d}\gamma(\boldsymbol{y}) - \mathcal{Q}_{G_{\mathrm{ev}}(\xi_n)} \langle \phi, v(\boldsymbol{y}) \rangle = \int_{\mathbb{R}^\infty} \langle \phi, v_{\mathrm{ev}}(\boldsymbol{y}) - \mathcal{I}^*_{G_{\mathrm{ev}}(\xi_n)} v_{\mathrm{ev}}(\boldsymbol{y}) \rangle \, \mathrm{d}\gamma(\boldsymbol{y}).$$

Hence, applying (3.37) in Theorem 3.10 for $p = 1$, we prove (ii):

$$\begin{aligned} \left| \int_{\mathbb{R}^\infty} \langle \phi, v(\boldsymbol{y}) \rangle \, \mathrm{d}\gamma(\boldsymbol{y}) - \mathcal{Q}_{G_{\mathrm{ev}}(\xi_n)} \langle \phi, v \rangle \right| &\le \int_{\mathbb{R}^\infty} \left| \langle \phi, v_{\mathrm{ev}}(\boldsymbol{y}) - \mathcal{I}^*_{G_{\mathrm{ev}}(\xi_n)} v_{\mathrm{ev}}(\boldsymbol{y}) \rangle \right| \mathrm{d}\gamma(\boldsymbol{y}) \\ &\le \int_{\mathbb{R}^\infty} \|\phi\|_{(X^1)'} \|v_{\mathrm{ev}}(\boldsymbol{y}) - \mathcal{I}^*_{G_{\mathrm{ev}}(\xi_n)} v_{\mathrm{ev}}(\boldsymbol{y})\|_{X^1} \, \mathrm{d}\gamma(\boldsymbol{y}) \\ &\le C \left\| v_{\mathrm{ev}} - \mathcal{I}^*_{G_{\mathrm{ev}}(\xi_n)} v_{\mathrm{ev}} \right\|_{\mathcal{L}_1(X^1)} \le C n^{-\min(\alpha,\beta)}. \end{aligned}$$

□

2.7. With the new corrected definition of $Q_{\Lambda_{\mathrm{ev}}(\xi_n)}$, the formulation of Corollaries 4.2 and 5.11 in Sections 4 and 5 of [1] is correct. But in the proofs the interpolation operator $I_{\Lambda_{\mathrm{ev}}(\xi_n)}$ is corrected as $I^*_{\Lambda_{\mathrm{ev}}(\xi_n)}$.

2.8. The interpolation operator $\mathcal{I}_{G_{\mathrm{ev}}(\xi)}$ in the formulation of Theorem 5.10 in Section 5 of [1] is corrected as $\mathcal{I}^*_{G_{\mathrm{ev}}(\xi)}$.

3. By the same argument, the interpolation operator $\mathcal{I}_{G_{\mathrm{ev}}(\xi)}$ in the formulation of Theorems 6.8 in Section 6 from [1] is corrected as $\mathcal{I}^*_{G_{\mathrm{ev}}(\xi)}$, and the formulation of Corollary 6.8 in Section 6 from [1] is correct.

4. The author would like to thank Jacob Zech for pointing out incorrectness of the integral approximation by the quadrature operator $Q_{\Lambda_{\mathrm{ev}}(\xi)}$ based on the old definition of operator Δ^Q_m in [1], page 1185, which leads in particular, to incorrectness of the proof of Corollary 4.2 from [1].

References

[1] D. Dũng, Sparse-grid polynomial interpolation approximation and integration for parametric and stochastic elliptic PDEs with lognormal inputs. *ESAIM: M2AN* **55** (2021) 1163–1198.

[2] D. Dũng, Sparse-grid polynomial interpolation approximation and integration for parametric and stochastic elliptic PDEs with lognormal inputs. Preprint arXiv:1904.06502v9 [math.NA] (2022).