ERRATUM TO "A C1-P2 FINITE ELEMENT WITHOUT NODAL BASIS"

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In [1], a finite element interpolation operator is defined and it is claimed (incorrectly) that the operator preserves the finite element functions (piecewise P_2 polynomials) and that consequently it preserves P_2 polynomials. In this note, we would change the definition of this interpolation operator, from (1) to (5) below, so that it preserves only P_2 polynomials.

In [1], a dual basis ψ_i, corresponding to a finite element nodal basis ϕ_i of $V_{h,0}$, is defined on a 9-square patch M_i as

$$\psi_i = \frac{583704}{553687}(\phi_{S_1} + \phi_{S_3} + \phi_{S_7} + \phi_{S_9}) - \frac{970452}{553687}(\phi_{S_2} + \phi_{S_4} + \phi_{S_6} + \phi_{S_8}) + \frac{1743594}{553687}\phi_{S_5}, \tag{1}$$

where S_l, cf. Figure 1, is the local indexing of squares around square Q_i ($i = S_5$). Then the local interpolation operator is defined by

$$I_h u = \sum_{i=1}^{n^2} u_i \phi_i, \quad \text{where} \quad u_i = \int_{M_i} \psi_i(\mathbf{x}) u(\mathbf{x}) d\mathbf{x}. \tag{2}$$

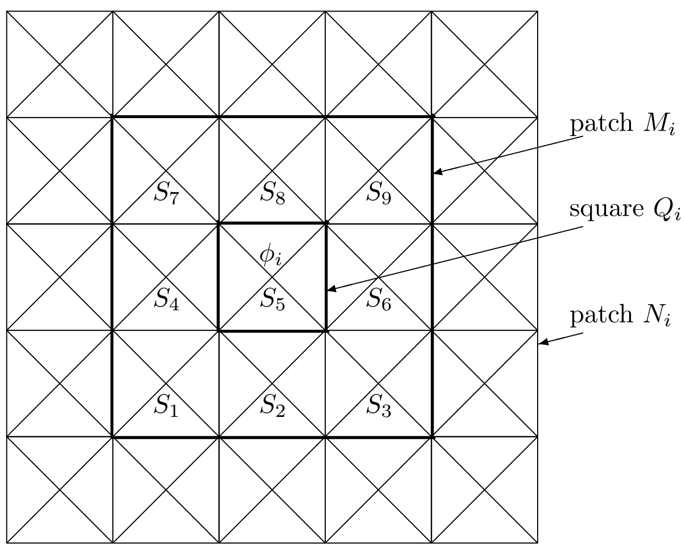

FIGURE 1. Local index mapping around one basis function ϕ_i.

Lemma 3.2 of [1] mistakenly claims that

$$I_h u = u \ \text{ on } Q_i \ \text{ if } \ u = \sum_{i=1}^{n^2} u_i \phi_i \text{ on } M_i, \tag{3}$$

and that consequently the interpolation preserves P_2 polynomials locally. Because of overlapping, one cannot have (3) but only have

$$I_h u = u \ \text{ on } Q_i \ \text{ if } \ u = \sum_{l=1}^{9} u_{S_l} \phi_{S_l} \text{ on } N_i, \tag{4}$$

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where N_i is a patch of 25 squares around Q_i, cf. Figure 1. Therefore, I_h does not preserve P_2 polynomials either.

We give up the preservation of all finite element $V_{h,0}$ functions. For preserving only P_2 polynomials locally, the dual basis function ψ_i is simply defined by, replacing (1),

$$\psi_i = 0(\phi_{S_1} + \phi_{S_3} + \phi_{S_7} + \phi_{S_9}) - \frac{45}{386}(\phi_{S_2} + \phi_{S_4} + \phi_{S_6} + \phi_{S_8}) + \frac{493}{772}\phi_{S_5}. \tag{5}$$

Lemma 3.2 of [1] is replaced by the next lemma.

Lemma 1. *If $u \in P_2(N_i)$, cf. Figure 1, then*

$$I_h u = u \quad \text{on } Q_i,$$

where I_h is defined in (2) with ψ_i defined in (5).

Proof. Let the lower-left corner of square Q_{S_1} be $(0,0)$, cf. Figure 1, and the grid size $h = 1$.

$$\int_{M_i} \psi_i(x,y)u(x,y)\,dx\,dy = \begin{cases} 1/2 & \text{if } u = 1, \\ 3/4 & \text{if } u = x, \text{ or } y, \\ 1 & \text{if } u = x^2, \text{ or } y^2, \\ 9/8 & \text{if } u = xy. \end{cases}$$

Combining 9 terms, with the definition of ϕ_{S_l}, we get $I_h x^k y^l = x^k y^l$ on Q_i for $k + l \leq 2$. ∎

Remark 1. *The rest analysis and results remain same in [1], after the above correction.* ∎

References

[1] S. Zhang, A C1-P2 finite element without nodal basis. M2AN Math. Model. Numer. Anal. 42 (2008), no. 2, 175–92.

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